

$$ds^2 = -dt^2 + a^2(t)dx^2 + a^2(t)dy^2 + a^2(t)dz^2$$

$$\Rightarrow ds ds = -dt dt + a^2(t)(dx dx + dy dy + dz dz)$$

i.e.,  $ds^2 = -dt^2 + a^2(t)\left(\sum_{i=1}^3 dx^i dx^i\right)$  i's are **not** powers.

The convention used here is this:  $dx^1 = dx$ ,  $dx^2 = dy$  &  $dx^3 = dz$

Now, I want to find the Christoffel symbols of the second kind and the Ricci tensors.

```
> restart;
> with(tensor) :
> coords := [t, x, y, z]:
> g := array(symmetric, sparse, 1..4, 1..4) :
> g[1, 1] := -1 :
> g[2, 2] := a(t)^2 :
> g[3, 3] := a(t)^2 :
> g[4, 4] := a(t)^2 :
> metric := create([-1,-1], eval(g));
```

$$metric := table \left( \begin{array}{l} index\_char = [-1, -1], \\ compts = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t)^2 & 0 & 0 \\ 0 & 0 & a(t)^2 & 0 \\ 0 & 0 & 0 & a(t)^2 \end{pmatrix} \end{array} \right) \quad (1)$$

```
> tensorsGR(coords, metric, contra_metric, det_met, C1, C2, Rm, Rc, R, GEin, C) :
> displayGR(Christoffel2, C2);
```

### The Christoffel Symbols of the Second Kind

non-zero components :

$$\{1,22\} = a(t) \left( \frac{d}{dt} a(t) \right)$$

$$\{1,33\} = a(t) \left( \frac{d}{dt} a(t) \right)$$

$$\{1,44\} = a(t) \left( \frac{d}{dt} a(t) \right)$$

$$\{2,12\} = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\{3,13\} = \frac{\frac{d}{dt} a(t)}{a(t)}$$

$$\{4,14\} = \frac{\frac{d}{dt} a(t)}{a(t)} \quad (2)$$

> `displayGR(Ricci, Rc);`

*The Ricci tensor  
non-zero components :*

$$R_{11} = \frac{3 \left( \frac{d^2}{dt^2} a(t) \right)}{a(t)}$$

$$R_{22} = -2 \left( \frac{d}{dt} a(t) \right)^2 - a(t) \left( \frac{d^2}{dt^2} a(t) \right)$$

$$R_{33} = -2 \left( \frac{d}{dt} a(t) \right)^2 - a(t) \left( \frac{d^2}{dt^2} a(t) \right)$$

$$R_{44} = -2 \left( \frac{d}{dt} a(t) \right)^2 - a(t) \left( \frac{d^2}{dt^2} a(t) \right)$$

*character : [-1, -1]*

**(3)**

So, it is fairly straightforward. Now what I wanted is this, instead of 4 dimensions (1 temporal, 3 spatial), I want to work with arbitrary dimensions, say  $n$ . In this case the above metric would be

$$ds^2 = -dt^2 + a^2(t) \left( \sum_{i=1}^{n-1} dx^i dx^i \right).$$

For the special case  $n=4$ , we have the above solutions.

**Now, in this generalized case, how can I find the Christoffel symbols of the second kind and the Ricci tensors?**

The main problem, seems to me is that I don't know how can I construct the array.

Is there any other method in Maple which can work with  $ds^2$  directly?

By the way, I don't want to give values to  $n$ , i.e. I want the answers in terms of  $n$ . **Not** for a particular value of  $n$ .