In[28]:=
```
ClearAll["Global`*"]
```

In[29]:=
```
doffactor[d_] = (d (d - 3))/2 + (d - 2);  (*factor related to the degrees of freedom*)
myH1[x_, d_, t_] = HankelH1[(d - 1)/2, x a[t]^-1];
myHP1[x_, d_, t_] = D[myH1[x, d, t], t];
myH2[x_, d_, t_] = HankelH2[(d - 1)/2, x a[t]^-1];
myHP2[x_, d_, t_] = D[myH2[x, d, t], t];
myBeta[x_, d_, t_] =
  -I ((Pi/(8 x))^(1/2)) a[t]^-((d - 1)/2) Exp[I x] (myHP2[x, d, t] + (-I x - (d - 1)/2) myH2[x, d, t]);
myBetastar[x_, d_, t_] = I ((Pi/(8 x))^(1/2)) a[t]^-((d - 1)/2) Exp[-I x]
   (myHP1[x, d, t] + (I x - (d - 1)/2) myH1[x, d, t]);
myBetasqr[x_, d_, t_] = Re[(myBeta[x, d, t]) (myBetastar[x, d, t])];
NumberDensity[x_, d_, t_] = myBetasqr[x, d, t]/((2 Pi)^(d - 1));
BE[x_] = 1/(Exp[x] - 1);
intfactor[x_, d_] = (2 ((Pi)^((d - 1)/2)))/Gamma[(d - 1)/2] (x^(d - 2));
(*factor related to the integration in d dimension*)
EnergyDensity[d_, t_] =
  NIntegrate[x (NumberDensity[x, d, t]) (BE[x]) (intfactor[x, d]), {x, 0, Infinity}];
```

```
NIntegrate::inumr : The integrand
  (2^(-1-d) π^(2+1/2 (-1+d)-d) x^(-1+d) Re[1/x a[t]^(1-d) (((1/2 Plus[≪2≫] + i x) ≪1≫ - ≪1≫) ((1/2 Plus[≪2≫] - i x)
      HankelH2[1/2 Plus[≪2≫], x Power[≪2≫]] - (x (HankelH2[≪1≫, ≪1≫] - ≪1≫) a′[t])/(2 a[t]^2)))]) / ((-
      1 + e^x) Gamma[1/2 (-1 + d)]) has evaluated to
   non-numerical values for all sampling points in the region with boundaries {{∞, 0.}}. ≫
```


```
NIntegrate::inumr : The integrand
  (2^(-1-d) π^(2+1/2 (-1+d)-d) x^(-1+d) Re[1/x a[t]^(1-d) (((1/2 Plus[≪2≫] + i x) ≪1≫ - ≪1≫) ((1/2 Plus[≪2≫] - i x)
      HankelH2[1/2 Plus[≪2≫], x Power[≪2≫]] - (x (HankelH2[≪1≫, ≪1≫] - ≪1≫) a′[t])/(2 a[t]^2)))]) / ((-
      1 + e^x) Gamma[1/2 (-1 + d)]) has evaluated to
   non-numerical values for all sampling points in the region with boundaries {{∞, 0.}}. ≫
```

In[41]:=
```
With[{d = 4},
 sol = NDSolve[{a'[t] == a[t] Sqrt[16 Pi/((d - 1) (d - 2)) EnergyDensity[d, t]], a[0] == 1}, a, {t, 0, 10}]]
```

NIntegrate::inumr :

The integrand $\frac{1}{16\left(-1+e^{x}\right)\pi} x^{3} \operatorname{Re}\left[1/\left(x\ a[t]^{3}\right)\left(\left(\left(-\frac{3}{2}+i\ x\right) \text{HankelH1}\left[\frac{3}{2}, x\ \text{Power}[\ll 2\gg]\right] - \frac{x\left(\text{HankelH1}\left[\frac{1}{2}, \text{Times}[\ll 2\gg]\right] - \ll 8\gg[\ll 1\gg]\right) a'[t]}{2\ a[t]^{2}}\right)\left(\left(-\frac{3}{2}-i\ x\right) \text{HankelH2}\left[\frac{3}{2}, x\ \text{Power}[\ll 1\gg]\right] - \frac{\ll 1\gg}{\ll 1\gg}\right)\right)\right]$ has evaluated to non−numerical values for all sampling points in the region with boundaries $\{\{\infty, 0.\}\}$. ≫

NIntegrate::inumr :

The integrand $\frac{1}{16\left(-1+e^{x}\right)\pi} x^{3} \operatorname{Re}\left[1/\left(x\ a[t]^{3}\right)\left(\left(\left(-\frac{3}{2}+i\ x\right) \text{HankelH1}\left[\frac{3}{2}, x\ \text{Power}[\ll 2\gg]\right] - \frac{x\left(\text{HankelH1}\left[\frac{1}{2}, \text{Times}[\ll 2\gg]\right] - \ll 8\gg[\ll 1\gg]\right) a'[t]}{2\ a[t]^{2}}\right)\left(\left(-\frac{3}{2}-i\ x\right) \text{HankelH2}\left[\frac{3}{2}, x\ \text{Power}[\ll 1\gg]\right] - \frac{\ll 1\gg}{\ll 1\gg}\right)\right)\right]$ has evaluated to non−numerical values for all sampling points in the region with boundaries $\{\{\infty, 0.\}\}$. ≫

NIntegrate::inumr :

The integrand $\frac{1}{16\left(-1+e^{x}\right)\pi} x^{3} \operatorname{Re}\left[1/\left(x\ a[t]^{3}\right)\left(\left(\left(-\frac{3}{2}+i\ x\right) \text{HankelH1}\left[\frac{3}{2}, x\ \text{Power}[\ll 2\gg]\right] - \frac{x\left(\text{HankelH1}\left[\frac{1}{2}, \text{Times}[\ll 2\gg]\right] - \ll 8\gg[\ll 1\gg]\right) a'[t]}{2\ a[t]^{2}}\right)\left(\left(-\frac{3}{2}-i\ x\right) \text{HankelH2}\left[\frac{3}{2}, x\ \text{Power}[\ll 1\gg]\right] - \frac{\ll 1\gg}{\ll 1\gg}\right)\right)\right]$ has evaluated to non−numerical values for all sampling points in the region with boundaries $\{\{\infty, 0.\}\}$. ≫

General::stop : Further output of NIntegrate::inumr will be suppressed during this calculation. ≫

NDSolve::ndnum : Encountered non−numerical value for a derivative at t == 0.`. ≫

NDSolve::ndnum : Encountered non−numerical value for a derivative at t == 0.`. ≫

Out[41]= NDSolve[

$\left\{a'[t] == 2\sqrt{\frac{2\pi}{3}}\ a[t]\ \sqrt{\text{NIntegrate}[x\ \text{NumberDensity}[x, 4, t]\ \text{BE}[x]\ \text{intfactor}[x, 4], \{x, 0, \infty\}]},\right.$

$\left. a[0] == 1\right\}$, a, {t, 0, 10}]