

$$\dot{a} = -\bar{\mu} a - \frac{\alpha_6}{4} a \sin \gamma \quad (3.9.19)$$

$$a \dot{\gamma} = 2 a \sigma - \frac{6}{8} \left(\alpha_1 - \alpha_2 + \frac{\alpha_3}{3} \right) a^3 - \frac{\alpha_6}{2} a \cos \gamma \quad (3.9.20)$$

By substituting $\dot{a} = 0$ and $\dot{\gamma} = 0$, one may note from the equation (3.9.19)-(3.9.20) that the system possess both trivial and nontrivial responses. Hence one may obtain the both responses by solving the equations (3.9.19)-(3.9.20) simultaneously.

In this case, to determine the stability of the steady state response system one may convert the polar form of modulations (i.e. equation (3.9.19) and (3.9.20)) into Cartesian form of modulation by letting $p = a \cos \gamma$ and $q = a \sin \gamma$. One may obtain following Cartesian form of modulations as

$$\dot{p} = -\mu p - 2\sigma q - \frac{1}{8}(p^2 + q^2)(\eta p - 6\kappa q) + \frac{1}{2}\alpha_4 \frac{pq}{(p^2 + q^2)^{\frac{1}{2}}} \quad (3.9.21)$$

$$\dot{q} = -\mu q - 2\sigma p - \frac{1}{8}(p^2 + q^2)(\eta q + 6\kappa p) - \frac{3}{4}\alpha_4 \frac{pq}{(p^2 + q^2)^{\frac{1}{2}}} \quad (3.9.22)$$

Hence, to obtain the stability of the steady state fixed-point response (p_0, q_0) , one may disturb the equilibrium point (p_0, q_0) by substituting $p = p_0 + p_1$, and $q = q_0 + q_1$, in equations (3.9.21) and (3.9.22) and finding the eigenvalues of the resulting Jacobean matrix (J). One can express the Jacobian matrix as follows

$$J = \begin{bmatrix} -\mu - \frac{3}{8}\eta p_0^2 + \frac{3}{2}p_0 \kappa q_0 - \frac{1}{8}\eta q_0^2 & -2\sigma - \frac{1}{4}q_0 \eta p_0 + \frac{9}{4}\kappa q_0^2 + \frac{3}{4}\kappa p_0^2 \\ + \frac{\alpha_4 p_0^2 q_0}{2\sqrt{p_0^2 + q_0^2}} - \frac{\alpha_4 p_0^2 q_0}{2(p_0^2 + q_0^2)^{\frac{3}{2}}} & + \frac{\alpha_4 p_0^2 q_0}{2\sqrt{p_0^2 + q_0^2}} - \frac{\alpha_4 p_0^2 q_0}{2(p_0^2 + q_0^2)^{\frac{3}{2}}} \\ -\mu - \frac{3}{8}\eta p_0^2 - \frac{3}{2}p_0 \kappa q_0 - \frac{1}{8}\eta q_0^2 & -2\sigma - \frac{1}{4}q_0 \eta p_0 - \frac{9}{4}\kappa q_0^2 - \frac{3}{4}\kappa p_0^2 \\ - \frac{3\alpha_4 p_0^2 q_0}{4\sqrt{p_0^2 + q_0^2}} + \frac{3\alpha_4 p_0^2 q_0}{4(p_0^2 + q_0^2)^{\frac{3}{2}}} & - \frac{3\alpha_4 p_0^2 q_0}{4\sqrt{p_0^2 + q_0^2}} + \frac{3\alpha_4 p_0^2 q_0}{4(p_0^2 + q_0^2)^{\frac{3}{2}}} \end{bmatrix} \quad (3.9.23)$$

In this resonance condition, the response of the system will be stable if and only if the real part of all the eigenvalues are negative.