$$f(\nu,u)=\frac{\sqrt{32}}{\pi}\sqrt{\cosh\nu-\cos u}\sum_{n=1}^{\infty}\Big(-n\,q_n(\nu)+A_n\,p_n(\nu)\Big)\sin nu$$

where

$$p_n(\nu)=P_{n-1/2}(\cosh\nu),\quad q_n(\nu)=Q_{n-1/2}(\cosh\nu)+\frac{i\pi}{2}P_{n-1/2}(\cosh\nu)$$

are the real combinations of half-integer Legendre functions for $\nu\geq 0$.

Differentiating this w.r.t. ν at $\nu=\nu_0$ corresponding to the surface of the torus and requiring this derivative to be zero yields the following infinite system of equations

$$\begin{pmatrix}\alpha_1 & -\beta_2 & 0 & 0 & 0 & \cdots\\ -\beta_1 & \alpha_2 & -\beta_3 & 0 & 0 & \cdots\\ 0 & -\beta_2 & \alpha_3 & -\beta_4 & 0 & \cdots\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots\end{pmatrix}\begin{pmatrix}A_1\\ A_2\\ A_3\\ \vdots\end{pmatrix}=\begin{pmatrix}\gamma_1-\delta_2\\ \gamma_2-\delta_3-\delta_1\\ \gamma_3-\delta_4-\delta_2\\ \vdots\end{pmatrix}$$

where

$$\begin{aligned}\alpha_n&=\sinh\nu_0\ p_n(\nu_0)+2\cosh\nu_0\ p_n'(\nu_0)\\ \beta_n&=p_n'(\nu_0)\\ \gamma_n&=n\ \sinh\nu_0\ q_n(\nu_0)+2n\ \cosh\nu_0\ q_n'(\nu_0)\\ \delta_n&=n\,q_n'(\nu_0)\end{aligned}$$

This seems hard, but it turns out that the coefficients A_n falls to zero rather quickly. The practical solution is to introduce some cutoff N so that the system of equations becomes finite

$$\begin{pmatrix}\alpha_1 & -\beta_2 & 0 & 0 & 0 & \cdots & 0\\ -\beta_1 & \alpha_2 & -\beta_3 & 0 & 0 & \cdots & 0\\ 0 & -\beta_2 & \alpha_3 & -\beta_4 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0 & 0 & -\beta_{N-1} & \alpha_N\end{pmatrix}\begin{pmatrix}A_1\\ A_2\\ A_3\\ \vdots\\ A_N\end{pmatrix}=\begin{pmatrix}\gamma_1-\delta_2\\ \gamma_2-\delta_3-\delta_1\\ \gamma_3-\delta_4-\delta_2\\ \vdots\\ \gamma_N-\delta_{N-1}\end{pmatrix}$$

and N is chosen so that the magintude of $A_N p_N(\nu_0)$ is below some threshold (this is because $0<\nu<\nu_0$ where $\nu=0$ corresponds to z axis and/or region far from the torus and ν_0 corresponds to the surface of the torus, moreover, $p_n(\nu)$ are strictly increasing functions; the value of N therefore serves as a global approximation for a given ν_0). The following table summarizes the maximum needed N for threshold equal to 10^{-6} for various values of $\kappa = r/c, \quad 0 < \kappa < 1$.

κ	ν_0	N
0.01	5.298	3
0.10	2.993	5
0.30	1.874	8
0.70	0.896	17
0.90	0.467	35
0.95	0.323	52
0.99	0.142	123

It is obvious that the larger κ is, the more (numerical) effort is needed to calculate field to a desired precision.

The relationship between torus' major radius c, minor (tube) radius r and parameters of hyperbolic coordinates a and ν_0 are

$$a = c\sqrt{1-\kappa^2}, \quad \nu_0 = \log\left(\frac{1}{\kappa} + \sqrt{\frac{1}{\kappa^2} - 1}\right)$$

where $\kappa = r/c$.

Once we have the solution f we can calculate $\vec{H}$ as $\vec{\nabla} f$.

To verify all of the above, I plotted the solution for $\vec{H}$ for various κ (= 0.1, 0.5, 0.9) and $c = 1$ in Mathematica as cuts in planes xz and xy. It shows both field streamlines (black solid lines) and field intensity (color blue = zero, white = 1 = far field, red = maximum (different for each κ)).

$\kappa = 0.1$: