MATH 1003, HK211 **Due Date:**____________ **Group (Names):** ____________________________

Project 08: Newton's Law of Cooling

Objective

To investigate Newton's Law of Cooling.

Narrative

Newton's Law of Cooling states that the rate dT/dt at which the temperature $T = T(t)$ of an object changes with respect to time t, is proportional to the difference $A - T$ between the ambient temperature A of the environment, and the temperature T of the object; that is

$$\frac{dT}{dt} = k(A - T) \qquad (*)$$

where $k > 0$ is a positive real constant.

Tasks

1. Using Maple, draw (in one graphic):

   a) the direction field associated to the differential equation for Newton's Law of Cooling assuming that $A = 80°$, $k = 0.5$, $t \in [0, 4]$, and $T \in [0, 125]$, and.

   b) the solutions to this equation that correspond to $T(0) = 10°$, $T(0) = 60°$, $T(0) = 120°$.

   At this point, make a hard-copy of your typed input and Maple's responses. Then, ...

2. On the graphic you produced for Task 1, label the coordinate axes, draw and label by hand the line whose equation is $T = A$, and label the curves corresponding to the three initial conditions. (Label the curve corresponding to $T(0) = 10°$ by "$T(0) = 10°$", for example.)

3. On the graphic you produced for Task 1, draw by hand the solution that corresponds to $T(0) = 100°$.

4. Use the curve you drew in Task 3 to estimate $T(4)$.

5. If $T(0) < A$:

   a) what does $(*)$ imply about the sign of dT/dt?

   b) does this mean T is increasing or decreasing?

   c) explain (on physical grounds) why T should approach A as t gets large.

6. If $T(0) > A$:

   a) what does $(*)$ imply about the sign of dT/dt?

   b) does this mean T is increasing or decreasing?

   c) explain (on physical grounds) why T should approach A as t gets large.