We have a system of coupled PDEs. One of the PDEs through separation of variables is converted to an ODE as follows:

$$w(x)\psi(y)\phi^2\xi - w^{(2)}(x)\phi^2\xi y + Gw(x)\psi_{,yy} - Nw^{(4)}(x)y + Nw^{(2)}(x)\psi(y) = 0$$

- The unknown functions are $w(x)$ and $\psi(y)$ (position shape functions). In addition, the parameter ϕ (time agent) takes positive real amounts. The goal is finding possible minimum amount of ϕ.
- The known parameters are ξ, N and G. In general, theses parameters are real functions in terms of y and in the domain of problem ($-K/2 \le y \le K/2$ and $K > 0$) are greater than zero. For simpler cases they are assumed as numeric constants.

Another PDE yields

$$\phi^2 = -\frac{1}{K}\int_{-K/2}^{K/2} \frac{G\psi_{,y}}{\xi} dy$$

B.C.s are ($V > 0$) :

$$\psi_{,y}(K/2\ ) = \psi_{,y}(-K/2) = 0$$

$$w(0) = w(V) = w^{(2)}(V) = 0$$

$$\psi_{,y}(0)\int wdx\Big|_{x=0} - w^{(1)}(0) = 0$$

In the last B.C., the constant amount arising from integration is taken to be zero.

1st method: (I don't know my method is mathematically logical or not!).

For the sake of convenience, the known parameters are assumed as the constant amounts. First, the unknown function w is assumed as an exponential function then the unknown function ψ is obtained.

$$w = \exp\left(\alpha\frac{x}{V}\right) \qquad 0 \le x \le V$$

$$\psi(y) = -\frac{\sin\left(\frac{\sqrt{V^2\phi^2\xi + N\alpha^2}\ y}{V\sqrt{G}}\right)\alpha^2\sqrt{G}}{\cos\left(\frac{1}{2}\frac{\sqrt{V^2\phi^2\xi + N\alpha^2}\ K}{V\sqrt{G}}\right)\sqrt{V^2\phi^2\xi + N\alpha^2}\ V} + \frac{\alpha^2 y}{V^2}$$

The unknown parameter α is obtained by substituting $w = \exp\left(\beta\frac{x}{V}\right)$ and ψ into 1st PDE. Multiplying resultant by y and integrating over the y, one has

$$\int_{-K/2}^{K/2}\left(\psi\phi^2\xi - \beta^2\phi^2\xi y + G\psi_{,yy} - N\beta^4 y + N\beta^2\psi\right)\times y\ dy = 0$$

It holds

$$\beta = \pm\alpha, \pm f(\alpha) \rightarrow w = c_1 e^{\alpha} + c_2 e^{-\alpha} + c_3 e^{f(\alpha)} + c_4 e^{-f(\alpha)}$$

where $f(\alpha)$ presents a function of α. The four boundary conditions for w yields to have four homogeneous equations. For nontrivial solution the determinant of known coefficients must be vanished. The resultant equation and the equation extracted from the second PDE, both are in terms of α and ϕ. Solving two mentioned equations simultaneously, yields the minimum possible amount of ϕ. It is noteworthy to mention that the PDEs are obtained in the conditions that the parameter ϕ takes possible minimum amount.

2st method:

We have two equations with further unknowns. The parameter ϕ must be minimized. Some optimization methods like the Spectral Ritz method may be applied to reach this goal. Is it possible to use MAPLE commands to minimize ϕ from two coupled nonlinear equations in the uploaded file **2.mw**?

Thank you for your attention