Assume an ODE of order three as follows:

$$y^{(3)}(x) = f\big(x, y(x), y'(x), y''(x)\big) \quad x \in [a, b]$$

in which $y^{(3)}(x)$ denotes third derivative of y with respect to x. At first step three sample points (i.e. $(x_n, f_n), (x_n - h, f_{n-1})$ and $(x_n - 2h, f_{n-2})$) are selected to interpolate the function $y^{(3)}(x)$,

$$y^{(3)}(x) = Ax^2 + Bx + C$$

where

$$A = \frac{1}{2h^2}(f_{n-2} - 2f_{n-1} + f_n)$$

$$B = \frac{1}{2h^2}(3hf_n + hf_{n-2} - 4hf_{n-1} - 2f_n x_n - 2f_{n-2}x_n + 4f_{n-1}x_n)$$

$$C = \frac{1}{2h^2}(2h^2 f_n - 3hf_n x_n - hf_{n-2}x_n + 4hf_{n-1}x_n + f_n x_n^2 + f_{n-2}x_n^2 - 2f_{n-1}x_n^2)$$

$$x_n = a + h \times n \quad (0 \le n \le N)$$

$$h = dx \approx \frac{b-a}{N}$$

$$f_n = f\big(x_n, y(x_n), y'(x_n), y''(x_n)\big)$$

The derivatives of y at x_n are approximated in terms of values of function y at some certain points.

$$y(x_n) = y_n$$

$$y'(x_n) = \frac{y_n - y_{n-1}}{h}$$

$$y''(x_n) = \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2}$$

Integrating from both sides of ODE yields

$$\int_{x_n}^{x_n+h}\int_{c_3}^{c_4}\int_{c_1}^{c_2} y^{(3)}(\xi)\,d\xi\,d\eta\,d\gamma = \int_{x_n}^{x_n+h}\int_{c_3}^{c_4}\int_{c_1}^{c_2} (A\xi^2 + B\xi + C)\,d\xi\,d\eta\,d\gamma$$

The parameters c_1 to c_4 are defined based on boundary condition. For example, when B.C. includes $y''(a), y'(a)$ the parameters c_1 to c_4 are equal to a, η, a and γ, respectively. Also, when B.C. includes $y''(a), y'(b)$ the parameters c_1 to c_4 are equal to a, η, γ and b, respectively. For last case, one has

$$(hy'(b) - y_{n+1} + y_n) - \left(bh - \frac{1}{2}(2x_n h + h^2)\right) y''(a) =$$

$$\frac{A}{60}((x_n+h)^5-(x_n)^5)+\frac{B}{24}((x_n+h)^4-(x_n)^4)+\frac{C}{6}((x_n+h)^3-(x_n)^3)-$$

$$\left(\frac{A}{3}a^3+\frac{B}{2}a^2+Ca\right)\left(bh-\frac{1}{2}(2x_nh+h^2)\right)$$

The above recursion equation gives the parameter y_{n+1} in n^{th} loop $(0\le n\le N,\ h^2\approx 0)$. It noteworthy to mention that, there exists another B.C. ($y_0=y(a)$ or $y_N=y(b)$). At the first loop some unknown parameters, y_{-1}, y_{-2} and y_{-3} are appeared that may be taken equal to y_0 for small amount of h. Approximating first and second derivatives of y with respect to x at boundary points, yields

$$y'(b)=\frac{y_N-y_{N-1}}{h}\rightarrow\ \ y_N=y'(b)h+y_{N-1}$$

$$y''(a)=\frac{y_1-2y_0+y_{-1}}{h^2}\ \ \rightarrow\ \ y_{-1}=y''(a)h^2-y_1+2y_0$$