PERGAMON

International Journal of Heat and Mass Transfer 43 (2000) 3253–3257

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Effect of transverse magnetic field on a flat plate thermometer

Ashok K. Singh[a], Pallath Chandran[b,*], Nirmal C. Sacheti[b]

[a]Department of Mathematics, Banaras Hindu University, Varanasi 221005, India
[b]Department of Mathematics and Statistics, College of Science, Sultan Qaboos University, Al Khod, PC 123, Muscat, Oman

Received 28 January 1998; received in revised form 19 May 1999

1. Introduction

The studies of boundary layer fluid flows and heat transfer over surfaces of different geometrical shapes are of great interest because of their applications in several industrial and physical fields. Of these, the flow over a flat plate, though classic in nature, has been investigated extensively in literature, and still continues to attract attention as an active area of research. The flow and heat transfer problems in this case often become amenable to similarity transformations, sometimes leading to even exact analytical results. Another advantage is that the momentum and energy equations may get decoupled so that these can be solved successively. It is also known that the boundary layer models developed for flow over a flat plate can also be adapted to typical practical applications, besides being helpful in understanding the basic dynamic features of the fluid flow. For instance, Chow [1] has discussed the flow of a viscous, heat conducting fluid over a flat plate, and has shown that the resulting thermal boundary layer problem can be used to model a flat plate thermometer which is mounted on a moving body such as, a flying aircraft.

In this paper, we have reconsidered the above thermometer problem with a view to investigating the effects of an externally applied magnetic field on the velocity and thermal boundary layers as well as on the wall parameters. As mentioned before, the hydromagnetic flow over a flat or vertical plate has been widely researched and reported in literature under several idealized assumptions, see, e.g., [2–6]. In continuation of these works, the present study concentrates on the energy aspect of the boundary layer flow, under an externally applied magnetic field. This necessitates consideration of the viscous and Ohmic dissipation terms in the energy equation which, in turn, adds to its degree of non-linearity.

In the next section, we have given a mathematical formulation of the thermometer problem in terms of two-dimensional steady state boundary layer equations with appropriate boundary conditions. Notably, problems of this type involve also derivative boundary conditions on the temperature variable, besides the usual conditions on the physical variables. The governing partial differential equations have been reduced to a system of ordinary differential equations using similarity transformations. The resulting non-linear boundary value problem has been solved numerically. Several case studies have been done and the effects of magnetic field on the flat plate thermometer problem have been discussed. Of particular interest in the present work is the influence of magnetic field on the shear stress and the plate temperature. From energy considerations, Chow [1] has shown that the plate temperature (also known as the *recovery factor*) is physically important for problems of this type. Furthermore, it had been shown in [1] that in the non-magnetic case, the recovery factor is less than unity for fluids whose Prandtl numbers, *Pr*, are below 1, and greater than unity for

* Corresponding author. Tel. +968-515-414.
E-mail address: chandran@squ.edu.om (P. Chandran).

PII: S0017-9310(99)00323-3

Nomenclature

B	magnetic field
c_p	specific heat at constant pressure
C_f	skin friction coefficient
H	Hartmann number
k	heat conductivity
M	magnetic parameter
Pr	Prandtl number
Re	Reynolds number
T	temperature
u	horizontal velocity
U	free stream velocity
v	vertical velocity

Greek symbols

η	dimensionless space variable
θ	dimensionless temperature
μ	viscosity coefficient
ν	kinematic viscosity
ρ	density
σ	electrical conductivity
τ	shear stress

Subscripts

1	free stream
w	plate

fluids whose Prandtl numbers are more than 1. Moreover, the recovery factor had been shown to be approximately equal to $Pr^{1/2}$ for Prandtl numbers close to unity, in agreement with an earlier conclusion by Pohlahausen [7]. The results of the present hydromagnetic study have, however, shown that the above conclusions are valid for viscous dissipation only. When Ohmic heating is present, it has been shown in Section 3 that the magnetic field tends to push up the plate temperature monotonically. The variations of the velocity and temperature profiles in the boundary layers have been shown for some typical fluids. The effects of magnetic field and heat conductivity on the thermal boundary layer have also been discussed.

2. Problem formulation

Consider the flow of an electrically conducting and viscous incompressible fluid past a semi-infinite flat plate under the influence of a transversely applied magnetic field. The x-axis is taken along the plate while y-axis is taken normal to it. Neglecting the induced magnetic field, the steady flow is governed by the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2}{\rho}(U-u) \tag{2}$$

with the boundary conditions

$$u=0, \quad v=0 \quad \text{at } y=0 \text{ and } u\to U \quad \text{as } y\to\infty \tag{3}$$

In Eqs. (1) and (2), u and v are the components of velocity in the x- and y-directions, respectively, ρ is the density, ν the kinematic viscosity, σ the electrical conductivity, U the free stream velocity and B the magnetic field strength.

The flat plate thermometer problem [1] in the presence of a transverse magnetic field can be studied by solving the energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2}{\rho c_p}u^2 \tag{4}$$

with the boundary conditions

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y=0 \quad \text{and } T\to T_1 \quad \text{as } y\to\infty, \tag{5}$$

in conjunction with Eqs. (1)–(3). In Eqs. (4) and (5), T is the temperature of the fluid near the boundary, T_1 is the temperature of the fluid in the free stream, k is the thermal conductivity and c_p is the specific heat at constant pressure. The second and third terms on the right-hand side of Eq. (4) arise due to viscous and Ohmic dissipations, respectively.

We now introduce the similarity transformations

$$\eta = y\sqrt{\frac{U}{\nu x}}, \quad u = Uf'(\eta), \quad v = \sqrt{\frac{\nu U}{4x}}(\eta f' - f),$$
$$M = \frac{\sigma x B^2}{\rho U}, \quad \theta = \frac{T-T_1}{U^2/(2c_p)}, \quad Pr = \frac{\nu\rho c_p}{k}, \tag{6}$$

In Eq. (6), M denotes the non-dimensional magnetic parameter which can also be expressed as H^2/Re, where H is the Hartmann number and Re is the Reynolds number, and Pr is the Prandtl number of the fluid. Using Eq. (6), it can be seen that the continuity equation (1) is automatically satisfied, while Eqs. (2) and (4) will be reduced, respectively, to

$$f''' + \frac{1}{2}ff'' + M(1 - f') = 0 \tag{7}$$

$$\theta'' + \frac{1}{2}Prf\theta' = -2Pr\left[(f'')^2 + M(f')^2\right]. \tag{8}$$

The boundary conditions are transformed into

$$f = 0, \quad f' = 0, \quad \theta' = 0 \quad \text{at}\ \eta = 0,$$

$$f' \to 1, \qquad \theta \to 0 \quad \text{as}\ \eta \to \infty. \tag{9}$$

In the above equations, prime denotes differentiation with respect to η.

Numerical solution of Eq. (7) with the appropriate boundary conditions has been obtained using a shooting method employing the Runge–Kutta algorithm. The values of M considered are 0.0, 0.02, 0.04 and 0.06. Eq. (8) with the relevant boundary conditions constitutes a boundary value problem involving a second-order ordinary differential equation with known coefficients. With the use of central finite difference formulae for first and second derivatives, Eq. (8) reduces to a set of equations in tridiagonal form. These equations have been solved by the Gaussian elimination method. A detailed description of the solution procedure is given in [1].

It should be noted that in order to satisfy the conditions on temperature as $\eta \to \infty$, a reasonably large η_{max} should be chosen. Numerical calculations have been carried out for mercury, air, sulphur dioxide and water, whose Prandtl numbers have been taken as 0.044, 0.71, 2.0 and 7.0, respectively. It has been observed through numerical experiments that for $M \neq 0$, suitable values of η_{max} are 65, 35, 23 and 14 corresponding to mercury, air, sulphur dioxide and water, respectively. For $M = 0$, it has been found that $\eta_{max} =$ 10 is suitable for the three fluids except mercury. For mercury, η_{max} had to be taken as large as 30. The results for these functions are presented in Section 3. We have also evaluated the skin friction and the plate temperature. The coefficient of skin friction C_f is given by

$$C_f = \frac{\tau}{\rho U^2} = \frac{f''(0)}{\sqrt{Re}} \tag{10}$$

where $\tau\,[= \mu(\partial u/\partial y)_{y=0}]$ is the shear stress at the plate and $Re\,[= Ux/\nu]$ is the Reynolds number.

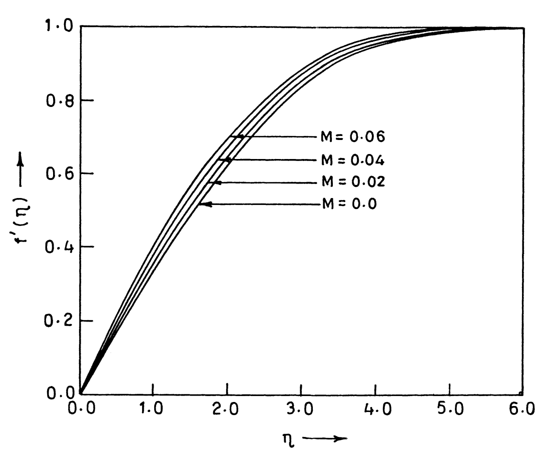

Fig. 1. Velocity profiles.

3. Results

The results of the numerical solution of Eqs. (7) and (8) have been presented in this section. Fig. 1 shows the variation of the dimensionless velocity u/U for different values of the magnetic parameter, while the variation of the dimensionless temperatures of mercury, air, sulphur dioxide and water have been shown in Figs. 2 and 3. Finally, the values of skin friction and plate temperature have been presented in Table 1.

As shown in Fig. 1, the variation of the horizontal velocity in the boundary layer follows the familiar pattern. It increases from zero on the plate to its free stream value through positive but decreasing gradients. As the magnetic field strength is increased, the velocity also increases in the boundary layer maintaining its concave down profile. For the relatively small values of the magnetic parameter considered here, it was seen that the velocity and the boundary layer thickness could be chosen in the neighbourhood of $\eta = 5.0$.

Regarding the temperature profiles, we observe from Figs. 2 and 3 that the convergence of their values to the corresponding free stream values are considerably influenced by the magnetic field. For illustration purposes, the temperature profiles have been shown for four different fluids: mercury, air, sulphur dioxide and water. These fluids exhibit decreasing heat conductivities in that order. As noted before, the numerical experiments showed that in the non-magnetic case the

Table 1
Skin friction $f''(0)$ and plate temperature $\theta(0)$

M	$f''(0)$	$\theta(0)$			
		$Pr = 0.044$	$Pr = 0.71$	$Pr = 2.0$	$Pr = 7.0$
0.00	0.33206	0.20125	0.84188	1.40742	2.52877
0.02	0.35884	0.42268	1.14112	1.72572	2.89756
0.04	0.38397	0.64153	1.44022	2.04313	3.26336
0.06	0.40771	0.88617	1.73206	2.35975	3.62653

dimensionless temperatures of these fluids converge to their free stream values for reasonably small values of η, ($\eta_{max} \approx 30$ for mercury and $\eta_{max} \approx 10$ for other fluids). This is in agreement with Chow's results [1]. However, even for small values of the magnetic parameter M, it was seen that one should increase η_{max} considerably, particularly for fluids of smaller Prandtl numbers. In the present case, η_{max} for mercury, air, sulphur dioxide and water had to be taken as large as 65, 35, 23 and 14, respectively, so as to yield satisfactory results. For all fluids considered, the magnetic field increases the temperature in the boundary layer. The rate of decrease of temperature to its free steam value is influenced both, by the heat conductivity of the fluid and the externally applied magnetic field. This decrease is much slower for mercury and air (cf. Fig. 2) than for sulphur dioxide and water (cf. Fig. 3). In fact, the curves for these latter fluids exhibit very large slopes, and it looks as if points of inflection would appear for fluids of higher Prandtl numbers. We thus see that the thermal boundary layer thickness increases with decreasing Prandtl number due to the increased conductivity of the fluid, which has also been observed in the non-magnetic case [1]. Furthermore, the magnetic field also aids in increasing the thermal boundary layer thickness. This effect of magnetic field in enhancing the boundary layer thickness is proportional to the heat conductivity of the fluid. For example, in our case

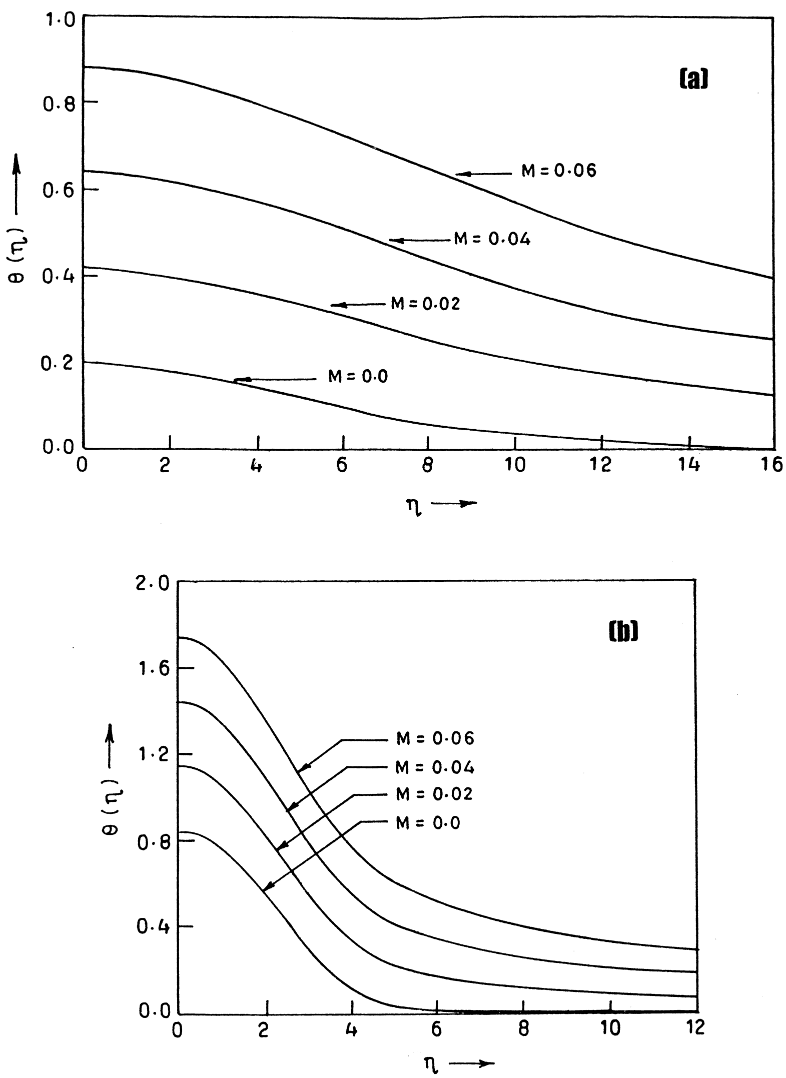

Fig. 2. Temperature profiles: (a) mercury ($Pr = 0.044$); (b) air ($Pr = 0.71$).

studies, mercury produced the largest thermal boundary layer thickness, while water had the smallest.

Finally, the skin friction $f''(0)$ and the plate temperature $\theta(0)$ are tabulated in Table 1. The skin friction increases with increase in the magnetic field. The effects of the magnetic field on the plate temperature are more revealing. As mentioned before, Chow [1] had shown that $\theta(0)$ is smaller or greater than unity with respect to Pr less or greater than 1, and $\theta(0)$ is approximately equal to $Pr^{1/2}$ in the non-magnetic case [7]. However, we see from Table 1 that $\theta(0)$ increases proportionally to both, the magnetic parameter and the Prandtl number. Its value can exceed unity for even fluids of Prandtl number less than 1, as can be seen from the results of air ($Pr = 0.71$). The plate temperatures of all fluids increase with increase in the magnetic field. This, in turn, would result in a corresponding increase in the thermal energy on the plate.

4. Conclusions

The combined effects of frictional forces and magnetic field on the thermal boundary layer near a flat plate have been considered in this note. The problem can be used to model a flat plate thermometer mounted on a moving body such as a flying aircraft. If the variation of temperature in the boundary layer is known, the ambient temperature T_1 can be calculated from the steady-state temperature T_w measured on the plate. The present work is an extension of a study in [1], and investigates the effect of an externally applied magnetic field on the temperature profiles. It has been shown that the magnetic field increases the velocity and temperature in the boundary layers. Moreover, the coupling between the heat conductivity of the fluid and the magnetic field acts additively to enhance the values of these field variables. The same is true with respect to the skin friction and plate temperature. It has also been shown that the dimensionless plate temperature can exceed unity for fluids of Prandtl number less than 1, which is at variance with available results for non-magnetic case.

References

[1] C.Y. Chow, An Introduction to Computational Fluid Mechanics, Wiley, New York, 1979, p. 242.

[2] V.J. Rossow, On flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field, NACA TR 1358 (1958).

[3] A.K. Singh, Hall effects on an oscillatory MHD flow in the Stokes problem past an infinite vertical porous plate, Astrophysics and Space Science 93 (1983) 1–13.

[4] O.P. Bhutani, P. Chandran, P. Kumar, Quasilinearised solution of axisymmetric, hydromagnetic stagnation point flow and heat transfer, Journal of Mathematical Analysis and Applications 98 (1984) 458–469.

[5] N.C. Sacheti, P. Chandran, A.K. Singh, An exact solution for unsteady magnetohydrodynamic free convection flow with constant heat flux, International Communications in Heat and Mass Transfer 21 (1994) 131–142.

[6] P. Chandran, N.C. Sacheti, A.K. Singh, Hydromagnetic flow and heat transfer past a continuously moving porous boundary, International Communications in Heat and Mass Transfer 23 (1996) 889–898.

[7] E. Pohlhausen, Der Warmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung, Zeitschrift für Angewandte Mathematik und Mechanik 1 (1921) 115.