Heat and Mass Transfer 40 (2004) 421–424
DOI 10.1007/s00231-003-0439-7

Effect of variable viscosity on boundary layer flow along a continuously moving plate with variable surface temperature

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Abstract Flow of an incompressible viscous fluid past a continuously moving semi-infinite plate is studied by taking into account variable viscosity and variable temperature. Velocity and temperature profiles are shown graphically whereas the numerical values of the skin-friction and the rate of heat transfer are listed in a table. The effect of different parameters on the flow field is discussed.

List of symbols

A	constant used in equation (6)
a	constant used in equation (1)
N	exponent used in equation (6)
C_f	skin-friction coefficient
f	dimensionless stream-function
k	thermal conductivity
Nu	local Nusselt number
Pr	Prandtl number
q	rate of heat transfer
Re	Reynolds number
T	temperature of fluid
T_r	reference temperature
T_∞	temperature of free stream
u, v	velocity components
U	velocity of the plate
x, y	Cartesian coordinates

Greek symbols

γ	a constant
τ	shear stress
η	similarity variable
θ	dimensionless temperature
θ_r	transformed dimensionless reference temperature
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
ψ	stream-function

Received: 29 October 2001
Published online: 28 January 2004

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Subscripts

w	condition at the wall
∞	property related to state

Superscripts

$'$	differentiation with respect to η

1 Introduction

Sakiadis (1961) first studied the flow of a viscous incompressible fluid past a semi-infinite continuously moving horizontal plate. Tsou et al. (1967) studied flow and heat transfer in the boundary layer on a continuously moving surface whereas Soundalgekar and Murty (1980) studied the heat transfer problem by assuming the plate temperature to be variable. In all these studies, the viscosity was assumed to be constant. However, viscosities of most of the fluids are temperature dependant and the polynomial model $\frac{1}{\mu} = \frac{1}{\mu_0}[1 + \gamma(T - T_\infty)]$ is a good approximation for most of the fluids. Here γ is a constant, μ_o is a reference viscosity and T, T_∞ are respectively the temperature of the fluid near and far away from the moving plate. This relation can be written as

$$\frac{1}{\mu} = a(T - T_r) \tag{1}$$

where $a = \frac{\gamma}{\mu_0}$ and $T_r = T_\infty - \frac{1}{\gamma}$.

Here a and T_r are assumed to be constant as their values are depending on the reference state and the thermal property of the fluid viz. γ. In general, $a > 0$ for liquids and $a < 0$ for gases.

The governing equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho_\infty C_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

All the physical variables are defined in the list of symbols.

The boundary conditions are

$$\left.\begin{aligned} y = 0, \quad u = U, \quad v = 0, \quad T = T_w(x) \\ y \to \infty, \quad u = 0, \quad T \to T_\infty \end{aligned}\right\} \tag{5}$$

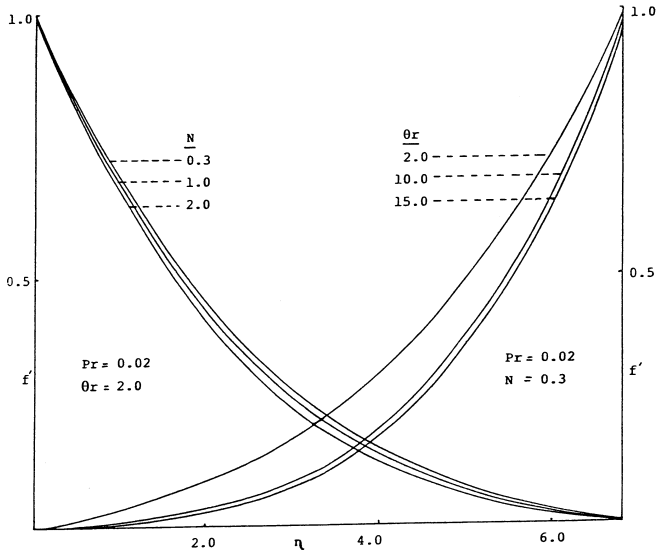

Fig. 1. Velocity profiles, Pr < 1.0, $\theta_r > 0.0$

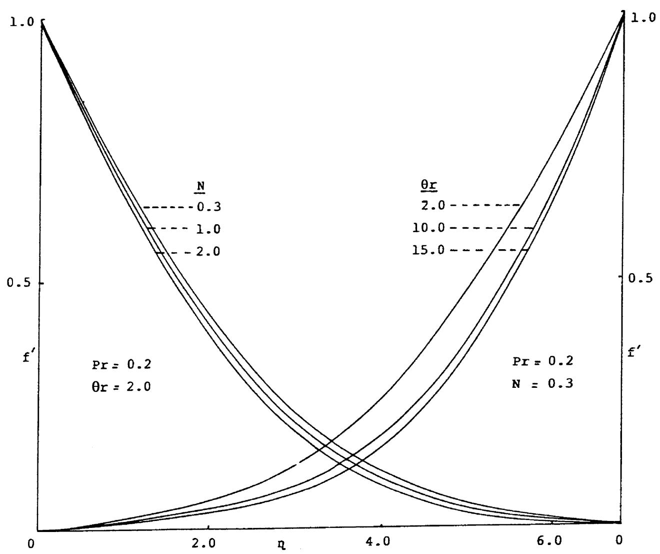

Fig. 2. Velocity profiles, Pr < 1.0, $\theta_r > 0.0$

We also assume the temperature of the plate to be varying as x^N and we take it as

$$T_w(x) - T_\infty = Ax^N \tag{6}$$

where A is a constant and x is measured from the leading edge.

Introducing the stream-function ψ and following similarity variables

$$\psi = v_\infty \mathrm{Re}^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{y}{x}\mathrm{Re}^{1/2},$$
$$\mathrm{Re} = \frac{Ux}{v_\infty}, \quad \mathrm{Pr} = \frac{\mu_\infty C_p}{k} \tag{7}$$

and carrying out the analysis, we can show that equations (2)–(6) in view of (7) reduce to

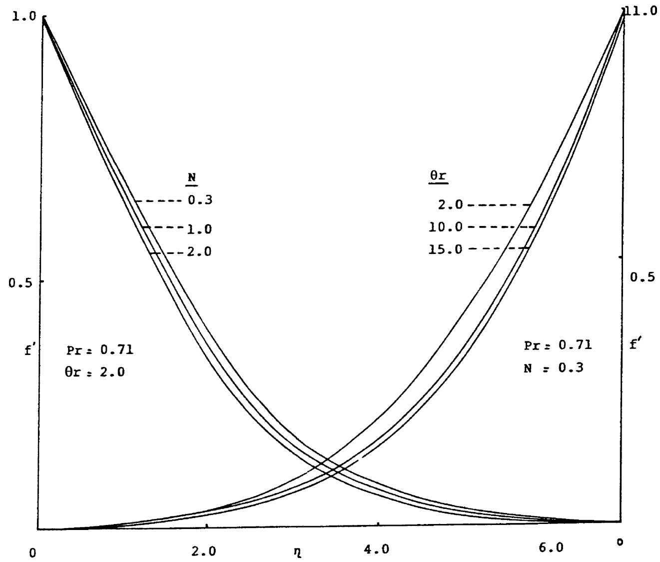

Fig. 3. Velocity profiles, Pr < 1.0, $\theta_r > 0.0$

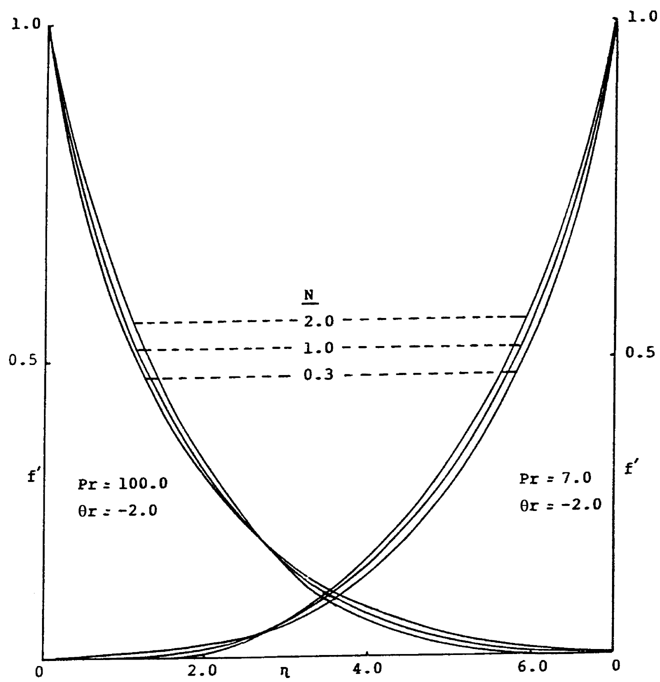

Fig. 4. Velocity profiles, Pr > 1.0, $\theta_r < 0.0$

$$f''' - \frac{1}{\theta - \theta_r}\theta' f'' - \frac{\theta - \theta_r}{2\theta_r} ff'' = 0 \tag{8}$$

$$\theta'' - N\mathrm{Pr} f'\theta + \frac{\mathrm{Pr}}{2} f\theta' = 0 \tag{9}$$

with following boundary conditions:

$$\left.\begin{array}{l} f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1 \quad \text{at } \eta = 0 \\ f'(\eta) = 0, \quad \theta(\eta) = 0 \quad \text{as } \eta \to \infty \end{array}\right\} \tag{10}$$

Here $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$ and $\frac{T - T_r}{T_w - T_r} = \theta - \theta_r$.

We now observe that θ_r is negative for liquids (Pr > 1.0) and positive for gases (Pr < 1.0). Equations (8)–(10) are

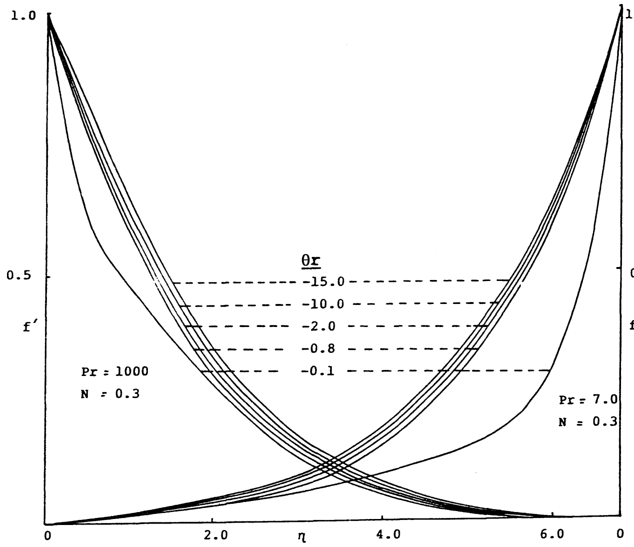

Fig. 5. Velocity profiles, Pr > 1.0, $\theta_r < 0.0$

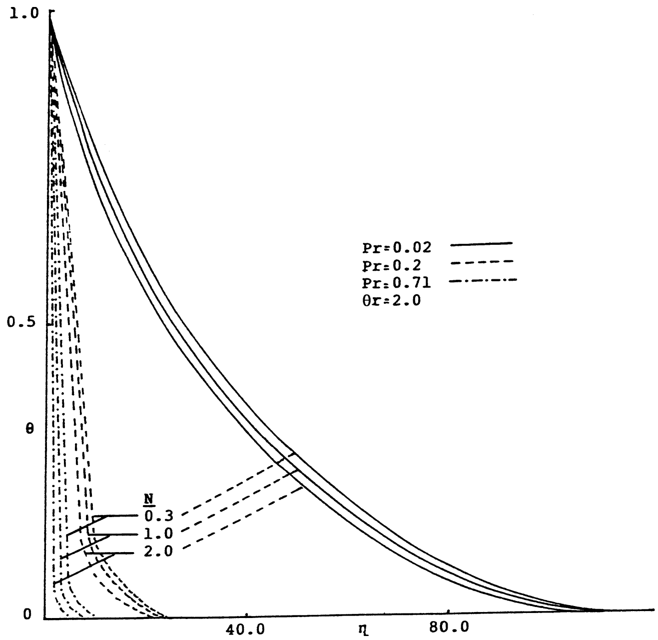

Fig. 6. Temperature profiles, Pr < 1.0, $\theta_r > 0.0$

solved numerically for Pr = 0.02, 0.2, 0.71, 7.0 and 100.0 and the velocity and temperature profiles are shown on graphs.

The skin friction in non-dimensional form is given by

$$C_f \mathrm{Re}^{1/2} = -\frac{2\theta_r}{1-\theta_r} f''(0, \theta_r) \tag{11}$$

where $C_f = \frac{2\tau_w}{\rho_\infty U^2}$.

The numerical values of $\{-f''(0)\}$ are computed and these are listed in Table 1.

The Nusselt number is given by

$$Nu \mathrm{Re}^{-1/2} = \{-\theta'(0, \theta_r)\} \tag{12}$$

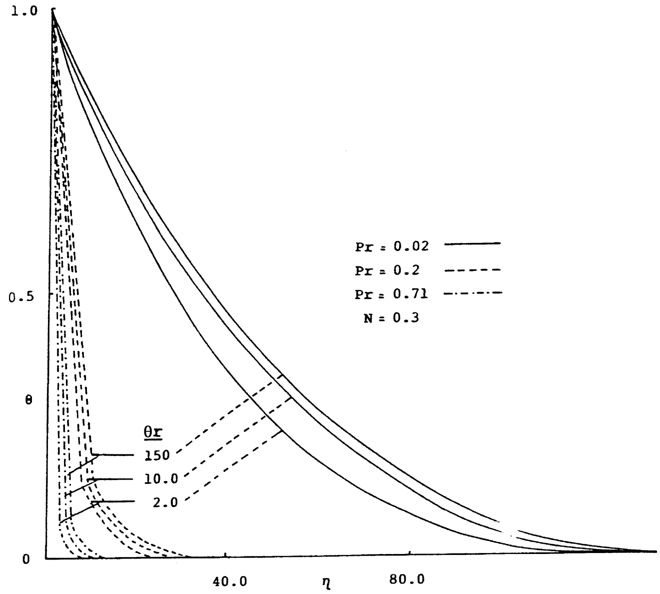

Fig. 7. Temperature profiles, Pr < 1.0, $\theta_r > 0.0$

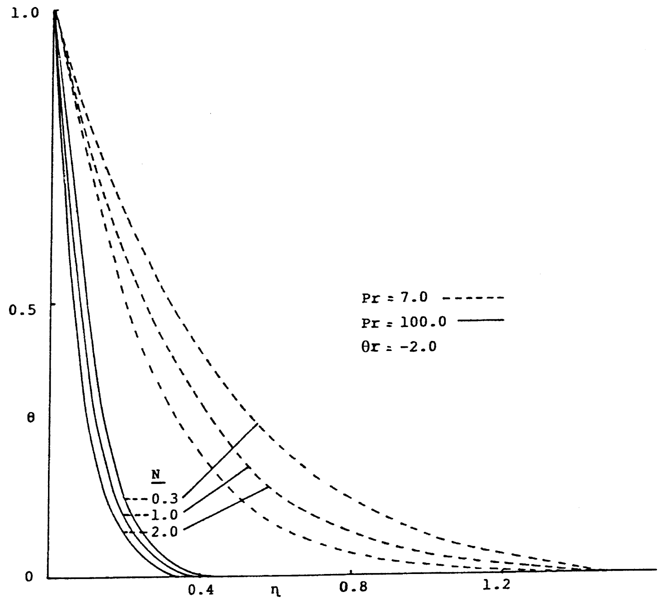

Fig. 8. Temperature profiles, Pr > 1.0, $\theta_r < 0.0$

where

$$Nu = \frac{xq_w}{k(T_w - T_r)}, \quad q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0}$$

The numerical values of $\{-\theta'(0)\}$ are computed and listed in Table 2.

2
Conclusions

(i) An increase in N, when θ_r is constant, leads to a decrease in the velocity. However, for N constant, when θ_r is increased, the velocity is also found to decrease but the effect of θ_r is more pronounced in case of a

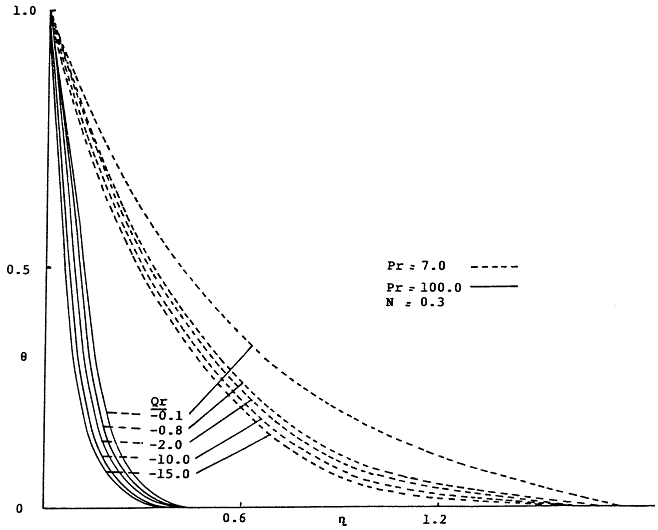

Fig. 9. Velocity profiles, Pr > 1.0, $\theta_r < 0.0$

Table 1. Values of $\{-f''(0)\}$

Pr	θ_r/N	0.0	0.3	1.0	2.0
0.02	2.0	0.3103	0.3091	0.3066	0.3036
	10.0	0.4206	0.4205	0.4202	0.4199
	15.0	0.4285	0.4284	0.4282	0.4280
0.2	2.0	0.2949	0.2906	0.2833	0.2765
	10.0	0.4187	0.4180	0.4166	0.4153
	15.0	0.4272	0.4267	0.4259	0.4250
0.71	2.0	0.2782	0.2730	0.2654	0.2591
	10.0	0.4156	0.4145	0.4127	0.4111
	15.0	0.4252	0.4245	0.4232	0.4222
7.0	−0.1	1.7399	1.8274	1.9984	2.1841
	−0.8	0.7940	0.8121	0.8394	0.8622
	−2.0	0.6029	0.6094	0.6186	0.6261
	−10.0	0.4781	0.4792	0.4808	0.4820
	−15.0	0.4668	0.4676	0.4686	0.4694
100.0	−0.1	2.8930	3.0556	3.3074	3.5232
	−0.8	0.9190	0.9292	0.9428	0.9530
	−2.0	0.6434	0.6464	0.6503	0.6532
	−10.0	0.4848	0.4853	0.4859	0.4863
	−15.0	0.4712	0.4715	0.4719	0.4722

fluid with very small Prandtl number. Due to the effect of *N*, the velocity of water and oil increases with an increase in the value of N for both Pr = 7.0 or 100.0. However, at small values of *N*, there appears a point of inflexion on the velocity profile. We conclude that the flow of water or oil may become unstable at small values of N. The velocity of water and oil is found to increase when values of $\{-\theta_r\}$ are increased.

(ii) The temperature in general decreases with increasing the Prandtl number for all values of N and $\{\pm\theta_r\}$. But for a particular value of $\theta_r = \pm 2.0$, the temperature is found to decrease with increasing *N* for all the fluids. But the effects of θ_r are different for gases and liquids. In case of gases, an increase in $\{\theta_r\}$ leads to an increase in the temperature of gases whereas an increase in $\{-\theta_r\}$ leads to a decrease in the temperature of liquids.

(iii) An increase in N leads to a fall in the values of $\{-f''(0)\}$ for gases and a rise in the values of $\{-f''(0)\}$ for liquids. Skin-friction is found to increase with increasing $\{\theta_r\}$ for gases and it decreases with increasing $\{\theta_r\}$ for liquids.

(iv) The Nusselt number increases with increasing *N* for all values of $\{\pm\theta_r\}$. We also observe that the Nusselt number increases with increasing the value of *N*, but an increase in $\{\theta_r\}$ leads to a decrease in the value of the Nusselt number for gases and when $\{-\theta_r\}$ is increased, the value of Nu is found to increase for liquids.

Table 2. Values of $\{-\theta'(0)\}$

Pr	θ_r/N	0.0	0.3	1.0	2.0
0.02	2.0	0.0294	0.0356	0.0627	0.0984
	10.0	0.0191	0.0288	0.0510	0.0813
	15.0	0.0188	0.0284	0.0502	0.0802
0.2	2.0	0.1569	0.2329	0.3812	0.5494
	10.0	0.1381	0.2090	0.3522	0.5188
	15.0	0.1368	0.2073	0.3500	0.5164
0.71	2.0	0.3807	0.5451	0.8440	1.1649
	10.0	0.3578	0.5191	0.8159	1.1367
	15.0	0.3561	0.5171	0.8138	1.1345
7.0	−0.1	1.1262	1.6283	2.5346	3.5021
	−0.8	1.3288	1.8602	2.7962	3.7881
	−2.0	1.3615	1.8969	2.8367	3.8309
	−10.0	1.3816	1.9192	2.8609	3.8561
	−15.0	1.3834	1.9212	2.8630	3.8583
100.0	−0.1	5.1404	7.1452	10.6598	14.3832
	−0.8	5.4739	7.5254	11.0901	14.8476
	−2.0	5.5154	7.5709	11.1389	14.8981
	−10.0	5.5389	7.5964	11.1660	14.9259
	−15.0	5.5409	7.5985	11.1683	14.9283

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