

```

> restart;
with(linalg):
with(LinearAlgebra):
with(Student[MultivariateCalculus]):
with( DynamicSystems ):
with(plots):
> M88:=Matrix(8, 8, {(1, 1) = 1, (1, 2) = 0, (1, 3) = 0, (1, 4) =
0, (1, 5) = 0, (1, 6) = 0, (1, 7) = 0, (1, 8) = 0, (2, 1) = 0,
(2, 2) = 1, (2, 3) = 0, (2, 4) = 0, (2, 5) = 0, (2, 6) = 0, (2,
7) = 0, (2, 8) = 0, (3, 1) = 0, (3, 2) = 0, (3, 3) = 1, (3, 4) =
0, (3, 5) = 0, (3, 6) = 0, (3, 7) = 0, (3, 8) = 0, (4, 1) = 0,
(4, 2) = 0, (4, 3) = 0, (4, 4) = 1, (4, 5) = 0, (4, 6) = 0, (4,
7) = 0, (4, 8) = 0, (5, 1) = 0, (5, 2) = 0, (5, 3) = 0, (5, 4) =
0, (5, 5) = 1, (5, 6) = 0, (5, 7) = 0, (5, 8) = 0, (6, 1) = 0,
(6, 2) = 0, (6, 3) = 0, (6, 4) = 0, (6, 5) = 0, (6, 6) = 1, (6,
7) = 0, (6, 8) = 0, (7, 1) = 0, (7, 2) = 0, (7, 3) = 0, (7, 4) =
0, (7, 5) = 0, (7, 6) = 0, (7, 7) = 1, (7, 8) = 0, (8, 1) = 0,
(8, 2) = 0, (8, 3) = 0, (8, 4) = 0, (8, 5) = 0, (8, 6) = 0, (8,
7) = 0, (8, 8) = 1});

```

$$M88 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(1)

```

>
> K88:=Matrix(8, 8, {(1, 1) = 2000000, (1, 2) = 0, (1, 3) = 0., (1,
4) = -1000000, (1, 5) = 0, (1, 6) = -99256.35650, (1, 7) = 0, (1,
8) = 0, (2, 1) = 0, (2, 2) = 2000000, (2, 3) = 0., (2, 4) = 0,
(2, 5) = -1000000, (2, 6) = 490049.1564, (2, 7) = 0, (2, 8) = 0,
(3, 1) = 0., (3, 2) = 0., (3, 3) = 19927.59766, (3, 4) =
-100000.0000, (3, 5) = 0.2051033808e-4, (3, 6) = -9925.635660,
(3, 7) = 0, (3, 8) = 0, (4, 1) = -1000000, (4, 2) = 0, (4, 3) =
-100000.0000, (4, 4) = 2000000, (4, 5) = 0, (4, 6) = 0., (4, 7) =
-1000000, (4, 8) = 0, (5, 1) = 0, (5, 2) = -1000000, (5, 3) =
0.2051033808e-4, (5, 4) = 0, (5, 5) = 2000000, (5, 6) = 0., (5,
7) = 0, (5, 8) = -1000000, (6, 1) = -99256.35650, (6, 2) =
490049.1564, (6, 3) = -9925.635660, (6, 4) = 0., (6, 5) = 0., (6,
6) = 500073.8114, (6, 7) = 99256.35650, (6, 8) = -490049.1564,
(7, 1) = 0, (7, 2) = 0, (7, 3) = 0, (7, 4) = -1000000, (7, 5) =
0, (7, 6) = 99256.35650, (7, 7) = 1000000, (7, 8) = 0, (8, 1) =
0, (8, 2) = 0, (8, 3) = 0, (8, 4) = 0, (8, 5) = -1000000, (8, 6)
= -490049.1564, (8, 7) = 0, (8, 8) = 2000000});

```

```

K88 := [[2000000, 0, 0., -1000000, 0, -99256.35650, 0, 0],
[0, 2000000, 0., 0, -1000000, 4.900491564 105, 0, 0],

```

(2)

```

[0., 0., 19927.59766, -1.000000000 105, 0.00002051033808, -9925.635660, 0, 0],
[-1000000, 0, -1.000000000 105, 2000000, 0, 0., -1000000, 0],
[0, -1000000, 0.00002051033808, 0, 2000000, 0., 0, -1000000],
[-99256.35650, 4.900491564 105, -9925.635660, 0., 0., 5.000738114 105, 99256.35650,
-4.900491564 105],
[0, 0, 0, -1000000, 0, 99256.35650, 1000000, 0],
[0, 0, 0, 0, -1000000, -4.900491564 105, 0, 2000000]]

```

```
> DD:=-MatrixMatrixMultiply(MatrixInverse(M88),K88);
```

```

DD := [[ -2.000000 106, -0., -0., 1.000000 106, -0., 99256.3565000000, -0., -0.],
        [-0., -2.000000 106, -0., -0., 1.000000 106, -4.90049156400000 105, -0., -0.],
        [-0., -0., -19927.5976600000, 1.00000 105, -0.0000205103380800000,
        9925.63566000000, -0., -0.],
        [1.000000 106, -0., 1.00000 105, -2.000000 106, -0., -0., 1.000000 106, -0.],
        [-0., 1.000000 106, -0.0000205103380800000, -0., -2.000000 106, -0., -0.,
        1.000000 106],
        [99256.3565000000, -4.90049156400000 105, 9925.63566000000, -0., -0.,
        -5.00073811400000 105, -99256.3565000000, 4.90049156400000 105],
        [-0., -0., -0., 1.000000 106, -0., -99256.3565000000, -1.000000 106, -0.],
        [-0., -0., -0., -0., 1.000000 106, 4.90049156400000 105, -0., -2.000000 106]]

```

(3)

```

> A:=blockmatrix(2,2,[ZeroMatrix(8),IdentityMatrix(8),DD,ZeroMatrix
(8)]);
A:=convert(A,Matrix);

```

```

A := [[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0],
        [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1],
        [-2.000000 106, 0., 0., 1.000000 106, 0., 99256.3565000000, 0., 0., 0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0., -2.000000 106, 0., 0., 1.000000 106, -4.90049156400000 105, 0., 0., 0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0., 0., -19927.5976600000, 1.00000 105, -0.0000205103380800000, 9925.63566000000,
        0., 0., 0, 0, 0, 0, 0, 0, 0, 0, 0],
        [0., 0., 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]

```

$$A := \left[ \begin{array}{l} 16 \times 16 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{array} \right] \quad (4)$$
[illegible]

$$B := \begin{bmatrix} 16 \times 1 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (5)$$

```
> C:=blockmatrix(1,2,[IdentityMatrix(8),ZeroMatrix(8)]);
C:=convert(C,Matrix);
```

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 8 \times 16 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (6)$$

```
> De:=Vector(<0,0,0,0,0,0,0,0,0,0>);
```

$$De := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

```
> sys := StateSpace(A,B,C,De);
```

```
> PrintSystem(sys);
```

## State Space

continuous

8 output(s); 1 input(s); 16 state(s)

inputvariable =  $[u1(t)]$

outputvariable =  $[y1(t), y2(t), y3(t), y4(t), y5(t), y6(t), y7(t), y8(t)]$

statevariable =  $[x1(t), x2(t), x3(t), x4(t), x5(t), x6(t), x7(t), x8(t), x9(t), x10(t), x11(t), x12(t), x13(t), x14(t), x15(t), x16(t)]$

a =  $\begin{bmatrix} 16 \times 16 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$

b =  $\begin{bmatrix} 16 \times 1 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$

c =  $\begin{bmatrix} 8 \times 16 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$

d =  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

**> TT:=TransferFunction(sys);**

**Transfer Function**

continuous

8 output(s); 1 input(s)

inputvariable =  $[uI(s)]$

outputvariable =  $[yI(s), y2(s), y3(s), y4(s), y5(s), y6(s), y7(s), y8(s)]$

**(9)**

**> PrintSystem(TT);**

**Transfer Function**

continuous

8 output(s); 1 input(s)

inputvariable =  $[uI(s)]$

outputvariable =  $[yI(s), y2(s), y3(s), y4(s), y5(s), y6(s), y7(s), y8(s)]$

$$tf_{1,1} = \frac{1.237685220 \cdot 10^{14} s^{10} + 8.051238112 \cdot 10^{20} s^8 + 1.553920831 \cdot 10^{27} s^6 + 8.8 \cdot 10^{34}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{2,1} = \frac{6.080061720 \cdot 10^{12} s^{10} + 4.876165479 \cdot 10^{19} s^8 + 1.224489214 \cdot 10^{26} s^6 + 9.8 \cdot 10^{33}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{3,1} = \frac{1.237685220 \cdot 10^{13} s^{10} + 1.050194446 \cdot 10^{20} s^8 + 3.143240602 \cdot 10^{26} s^6 + 3.9 \cdot 10^{33}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{4,1} = \frac{1.250000000 \cdot 10^8 s^{12} + 1.065000176 \cdot 10^{15} s^{10} + 3.208722438 \cdot 10^{21} s^8 + 4.063526632 \cdot 10^{28}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{5,1} = \frac{-2.53853423 \cdot 10^8 s^8 - 1.646277469 \cdot 10^{15} s^6 - 3.662044651 \cdot 10^{22}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{6,1} = \frac{-3.10176114 \cdot 10^8 s^{12} - 3.107942204 \cdot 10^{15} s^{10} - 1.122194723 \cdot 10^{22} s^8 - 1.754275254 \cdot 10^{29}}{3125 \cdot s^{16} + 3.600000440 \cdot 10^{10} s^{14} + 1.600621320 \cdot 10^{17} s^{12} + 3.450430710 \cdot 10^{23} s^{10} + 3.727040845 \cdot 10^{29}}$$

$$tf_{7,1} = \frac{125 \cdot s^{14} + 1.315000176 \cdot 10^9 s^{12} + 5.213716584 \cdot 10^{15} s^{10} + 9.665345752 \cdot 10^{21} s^8 + 8.496094 \cdot 10^{28}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

$$tf_{8,1} = \frac{-6.080061720 \cdot 10^{12} s^{10} - 4.876165480 \cdot 10^{19} s^8 - 1.224489214 \cdot 10^{26} s^6 - 9.8 \cdot 10^{33}}{125 \cdot s^{16} + 1.440000176 \cdot 10^9 s^{14} + 6.402485280 \cdot 10^{15} s^{12} + 1.380172284 \cdot 10^{22} s^{10} + 1.490816338 \cdot 10^{28} s^8}$$

**> MagnitudePlot(TT,linearfreq=true,linearmag = true,decibels = false,hertz = true,range = 0 .. 100,view=[0 .. 100,0 .. 0.0001]);**  
**#,range = 300 .. 1000,view=[300 .. 1000,0 .. 0.0001]**

