```
restart
with(plots):
```

# How can I find the surface area of a normal chicken egg?

There is a nice equation describing the equation of an egg curve credit to Nobuo Yamamoto :

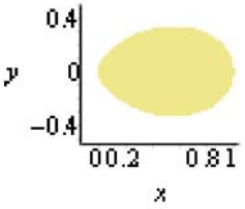
the following equation giving egg-like shaped curves.

$$(x^2+y^2)^2 = a\cdot x^3 + (a-b)\cdot x\cdot y^2$$

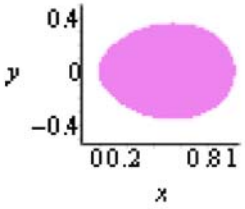
$$(x^2+y^2)^2 = a\,x^3 + (a-b)\,x\,y^2 \qquad \textbf{(1)}$$

where $a \geqslant b \geqslant 0$

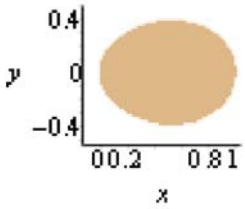
```
implicitplot((1)|a=1, b=1, x=0..1, y=-1..1, color="Khaki", view=[-0.1..1.05, -0.5..0.5], scaling=constrained, gridrefine=3, axes=framed, filledregions=true, coloring=["Khaki", white])
```




```
implicitplot((1)|a=1, b=0.8, x=0..1, y=-1..1, color="Violet", view=[-0.1..1.05, -0.5..0.5], scaling=constrained, gridrefine=3, axes=framed, filledregions=true, coloring=["Violet", white])
```




```
implicitplot((1)|a=1, b=0.6, x=0..1, y=-1..1, color="Burlywood", view=[-0.1..1.05, -0.5..0.5], scaling=constrained, gridrefine=3, axes=framed, filledregions=true, coloring=["Burlywood", white])
```




In the case of b=0.7*$a$ the curve approaches the shape of an actual egg most.

$$(x^2+y^2)^2 = a\cdot x^3 + (a-b)\cdot x\cdot y^2 \Big|_{b=\frac{7}{10}\cdot a}$$

$$(x^2+y^2)^2 = a\,x^3 + \frac{3}{10}\,a\,x\,y^2 \qquad \textbf{(2)}$$

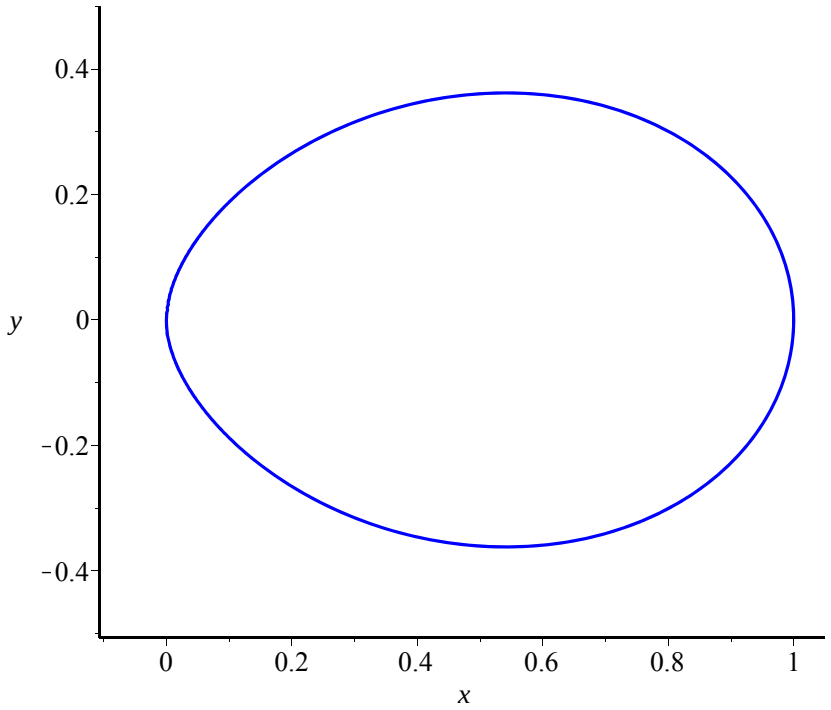
$$egg := (x^2+y^2)^2 = a\cdot x^3 + \frac{3\cdot a}{10}\,x\cdot y^2$$

$$(x^2+y^2)^2 = a\,x^3 + \frac{3}{10}\,a\,x\,y^2 \qquad \textbf{(3)}$$

where $0 \leq x \leq a$ , $a$ is the length of the major axis of symmetry for an egg.
the curve looks like the following for a=1 :

```
implicitplot(egg|a=1, x=0..1, y=-1..1, color=blue, view=[-0.1..1.05, -0.5..0.5], scaling
```

$= constrained, gridrefine = 3\ , axes = framed\Big)$




First we solve for y in (1) when y>0 :

$y1 := (solve(egg, y) \text{ assuming } y > 0)[1]$

$$\frac{1}{10}\sqrt{5\,x\sqrt{9\,a^2+280\,a\,x}+15\,a\,x-100\,x^2} \tag{4}$$

The derivative is:
$diff\,(y1, x)$

$$\frac{1}{20}\,\frac{5\sqrt{9\,a^2+280\,a\,x}+\dfrac{700\,x\,a}{\sqrt{9\,a^2+280\,a\,x}}+15\,a-200\,x}{\sqrt{5\,x\sqrt{9\,a^2+280\,a\,x}+15\,a\,x-100\,x^2}} \tag{5}$$

The formula of computing surface area by revolution is:

$$2\,\pi\int_0^a y\sqrt{1+\left(\frac{d}{d\,x}\,y\right)^2}\,dx:$$

Using the surface by revolution formula , we have:

S≈2.042087.

$$2\,\mathrm{Pi}\cdot\int_0^a y1\cdot\sqrt{1+diff\left(y1,x\right)^2}\,\mathrm{d}x\Bigg|_{a=1}$$

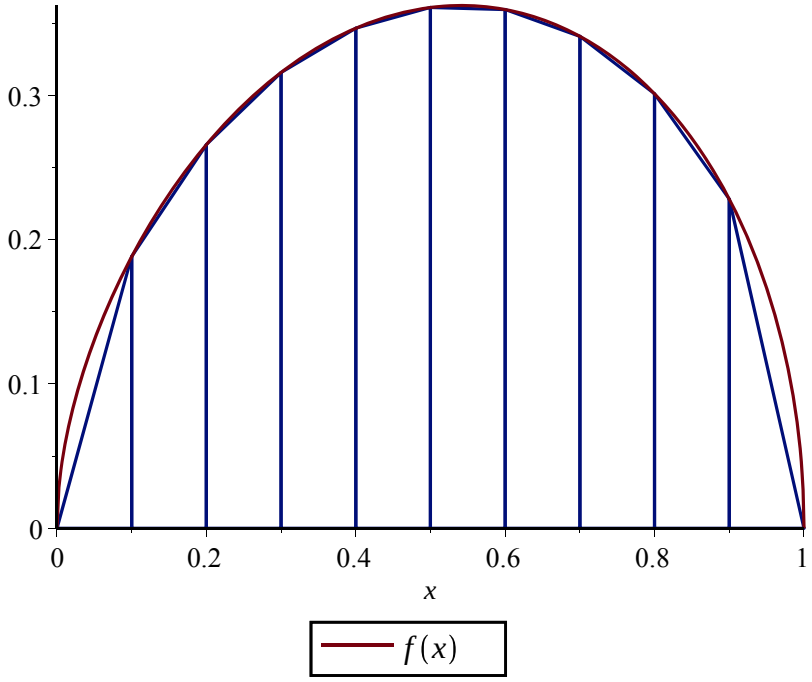
$$2\,\pi\left(\int_0^1 \frac{1}{200}\sqrt{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}\,\sqrt{400+\frac{\left(5\sqrt{280\,x+9}+\frac{700\,x}{\sqrt{280\,x+9}}+15-200\,x\right)^2}{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}}\,\mathrm{d}x\right)\xrightarrow{\text{at 5 digits}}$$

$$2.0422 \tag{7}$$

$$with(Student[NumericalAnalysis]):\ a:=1$$

$$1 \tag{8}$$

$$Quadrature(\mathrm{y1},\ \mathrm{x}=0..a,\ \mathrm{method}=\mathrm{trapezoid},\ \mathrm{partition}=10,\ \mathrm{output}=\mathrm{plot})$$



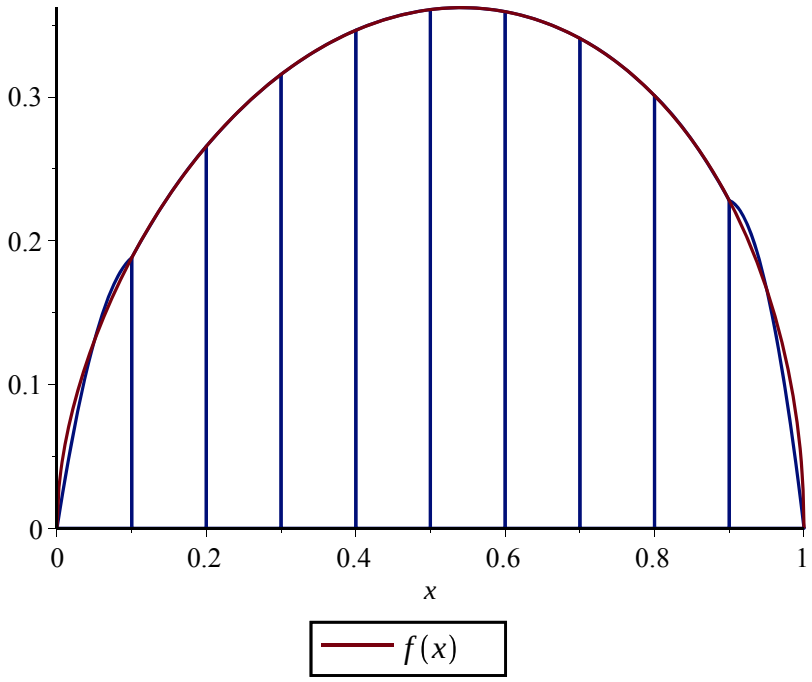

An approximation of the integral of

$\frac{1}{10}\sqrt{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}$ on the interval [0., 1.] using the trapezoid rule. Integral Value: 0.2793072219. Approximation: 0.2705899623.

*Quadrature*(y1, x = 0 ..a, method = trapezoid, partition = 10, output = information)

```
INTEGRAL: Int(1/10*(5*x*(280*x+9)^(1/2)+15*x-100*x^2)^(1/2),x=0.
.1) = 0.279307222
APPROXIMATION METHOD: Trapezoidal Rule
---------------------------------- INFORMATION TABLE ------------
----------------------
    Approximate Value         Absolute Error         Relative
Error
         0.270589962             0.0087172596             3.121 %
-----------------------------------------------------------------
----------------------
Number of Function Evaluations:     11
```

*Quadrature*(y1, x = 0. .*a*, method = simpson, partition = 10, *output* = *plot*)



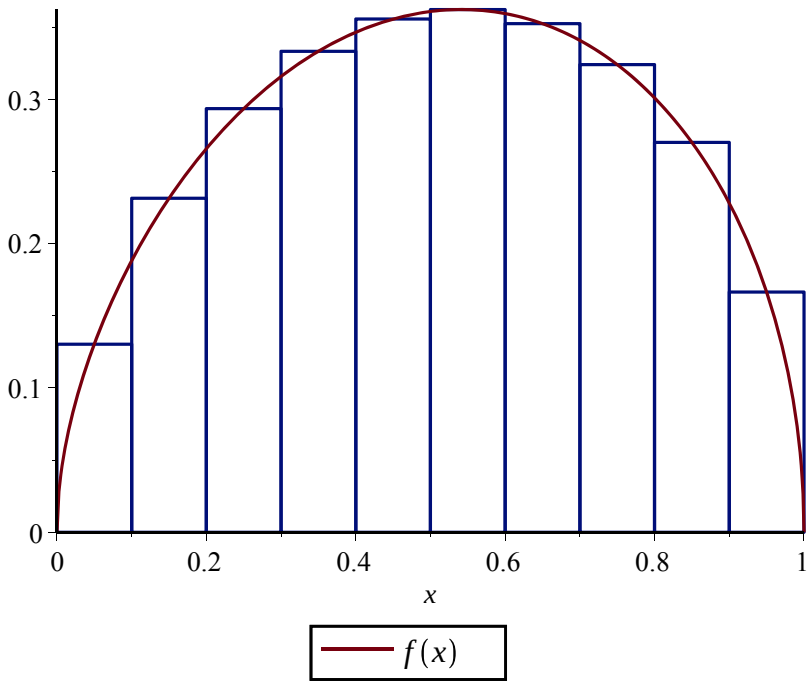

An approximation of the integral of

$\frac{1}{10}\sqrt{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}$ on the interval [ 0., 1.] using Simpson's rule. Integral Value: 0.2793072219. Approximation: 0.2781211381.

*Quadrature*( y1, x = 0. .*a*, method = simpson, partition = 10, output = information)

```
INTEGRAL: Int(1/10*(5*x*(280*x+9)^(1/2)+15*x-100*x^2)^(1/2),x=0.
.1) = 0.279307222
APPROXIMATION METHOD: Simpson's Rule
---------------------------------- INFORMATION TABLE ------------
----------------------
    Approximate Value         Absolute Error         Relative
Error
         0.278121138             0.0011860839             0.4247 %
-----------------------------------------------------------------
----------------------
Number of Function Evaluations:     21
```

*Quadrature*( y1, x = 0 ..a, method = midpoint, partition = 10, output = *plot*)

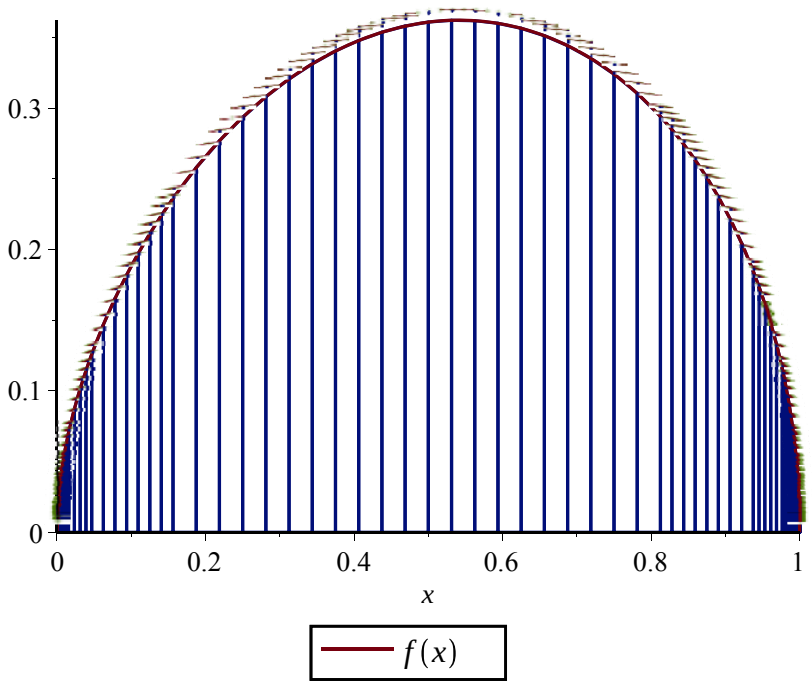



An approximation of the integral of
$\frac{1}{10}\sqrt{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}$ on the interval [ 0., 1. ] using a midpoint Riemann sum. Integral Value: 0.2793072219. Approximation: 0.2818867259.

*Quadrature*( y1, x = 0 ..a, method = midpoint, partition = 10, output = information)

```
INTEGRAL: Int(1/10*(5*x*(280*x+9)^(1/2)+15*x-100*x^2)^(1/2),x=0.
.1) = 0.279307222
APPROXIMATION METHOD: Midpoint Rule
---------------------------------- INFORMATION TABLE ------------
---------------------
    Approximate Value         Absolute Error         Relative
Error
         0.281886726             0.0025795039             0.9235 %
-----------------------------------------------------------------
---------------------
Number of Function Evaluations:     10
```

*AdaptiveQuadrature*( *y1*, *x* = 0 ..*a*, *method* = *trapezoid*, *output* = *plot*)




An approximation of the integral of

$\frac{1}{10}\sqrt{5\,x\sqrt{280\,x+9}+15\,x-100\,x^2}$ on the interval [ 0., 1. ] using adaptive trapezoid rule. Integral Value: 0.2793072219. Approximation: 0.2792691085.

*AdaptiveQuadrature*(*y1*, $x=0..a$, *method* = *trapezoid*, *output* = information)

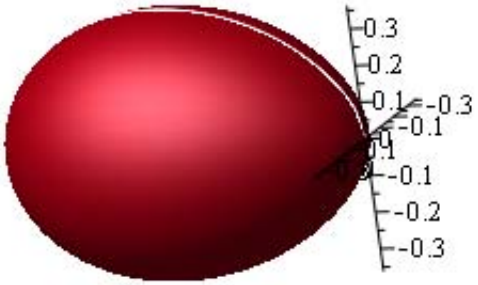
```
INTEGRAL: Int(1/10*(5*x*(280*x+9)^(1/2)+15*x-100*x^2)^(1/2),x=0.
.1) = 0.279307222
APPROXIMATION METHOD: Adaptive Trapezoidal Rule
---------------------------------- INFORMATION TABLE ------------
----------------------
    Approximate Value         Absolute Error         Relative
Error
         0.279269108             3.81134e-05           0.01365 %
---------------------------------- ITERATION HISTORY ------------
---------------------------
   Interval           Status         Present Stack
      0..1             fail           EMPTY
      0..1/2           fail           [1/2, 1]
      0..1/4           fail           [[1], [1/4, 1/2]]
      0..1/8           fail           [[2], [1/8, 1/4]]
      0..1/16          fail           [[3], [1/16, 1/8]]
      0..1/32          fail           [[4], [1/32, 1/16]]
      0..1/64          fail           [[5], [1/64, 1/32]]
      0..1/128         fail           [[6], [1/128, 1/64]]
      0..1/256         fail           [[7], [1/256, 1/128]]
      0..1/512         fail           [[8], [1/512, 1/256]]
      0..1/1024        fail           [[9], [1/1024, 1/512]]
```

```
      0..1/2048        fail          [[10], [1/2048, 1/1024]]
      0..1/4096        fail          [[11], [1/4096, 1/2048]]
      0..1/8192        fail          [[12], [1/8192, 1/4096]]
      0..1/16384        fail          [[13], [1/16384, 1/8192]]
      0..1/32768        fail          [[14], [1/32768, 1/16384]]

      0..1/65536        fail          [[15], [1/65536, 1/32768]]

      0..1/131072        PASS          [[16], [1/131072,
1/65536]]
 1/131072..1/65536        PASS          [[15], [1/65536,
1/32768]]
 1/65536..1/32768        PASS          [[14], [1/32768, 1/16384]
]
 1/32768..1/16384        PASS          [[13], [1/16384, 1/8192]]

 1/16384..1/8192        PASS          [[12], [1/8192, 1/4096]]
 1/8192..1/4096        PASS          [[11], [1/4096, 1/2048]]
 1/4096..1/2048        PASS          [[10], [1/2048, 1/1024]]
 1/2048..1/1024        PASS          [[9], [1/1024, 1/512]]
 1/1024..1/512        PASS          [[8], [1/512, 1/256]]
  1/512..1/256        fail          [[7], [1/256, 1/128]]
  1/512..3/1024        PASS          [[8], [3/1024, 1/256]]
 3/1024..1/256        PASS          [[7], [1/256, 1/128]]
  1/256..1/128        fail          [[6], [1/128, 1/64]]
  1/256..3/512        PASS          [[7], [3/512, 1/128]]
  3/512..1/128        PASS          [[6], [1/128, 1/64]]
  1/128..1/64        fail          [[5], [1/64, 1/32]]
  1/128..3/256        PASS          [[6], [3/256, 1/64]]
  3/256..1/64        PASS          [[5], [1/64, 1/32]]
   1/64..1/32        fail          [[4], [1/32, 1/16]]
   1/64..3/128        PASS          [[5], [3/128, 1/32]]
  3/128..1/32        PASS          [[4], [1/32, 1/16]]
   1/32..1/16        fail          [[3], [1/16, 1/8]]
   1/32..3/64        fail          [[4], [3/64, 1/16]]
   1/32..5/128        PASS          [[5], [5/128, 3/64]]
  5/128..3/64        PASS          [[4], [3/64, 1/16]]
   3/64..1/16        PASS          [[3], [1/16, 1/8]]
   1/16..1/8        fail          [[2], [1/8, 1/4]]
   1/16..3/32        fail          [[3], [3/32, 1/8]]
   1/16..5/64        PASS          [[4], [5/64, 3/32]]
   5/64..3/32        PASS          [[3], [3/32, 1/8]]
   3/32..1/8        fail          [[2], [1/8, 1/4]]
   3/32..7/64        PASS          [[3], [7/64, 1/8]]
   7/64..1/8        PASS          [[2], [1/8, 1/4]]
    1/8..1/4        fail          [[1], [1/4, 1/2]]
    1/8..3/16        fail          [[2], [3/16, 1/4]]
    1/8..5/32        fail          [[3], [5/32, 3/16]]
    1/8..9/64        PASS          [[4], [9/64, 5/32]]
   9/64..5/32        PASS          [[3], [5/32, 3/16]]
   5/32..3/16        PASS          [[2], [3/16, 1/4]]
   3/16..1/4        fail          [[1], [1/4, 1/2]]
   3/16..7/32        PASS          [[2], [7/32, 1/4]]
   7/32..1/4        PASS          [[1], [1/4, 1/2]]
    1/4..1/2        fail          [1/2, 1]
    1/4..3/8        fail          [[1], [3/8, 1/2]]
    1/4..5/16        fail          [[2], [5/16, 3/8]]
```

```
  1/4..9/32          PASS          [[3], [9/32, 5/16]]
  9/32..5/16         PASS          [[2], [5/16, 3/8]]
  5/16..3/8          fail          [[1], [3/8, 1/2]]
  5/16..11/32        PASS          [[2], [11/32, 3/8]]
 11/32..3/8          PASS          [[1], [3/8, 1/2]]
   3/8..1/2          fail          [1/2, 1]
   3/8..7/16         fail          [[1], [7/16, 1/2]]
   3/8..13/32        PASS          [[2], [13/32, 7/16]]
 13/32..7/16         PASS          [[1], [7/16, 1/2]]
  7/16..1/2          fail          [1/2, 1]
  7/16..15/32        PASS          [[1], [15/32, 1/2]]
 15/32..1/2          PASS          [1/2, 1]
   1/2..1            fail          EMPTY
   1/2..3/4          fail          [3/4, 1]
   1/2..5/8          fail          [[1], [5/8, 3/4]]
   1/2..9/16         fail          [[2], [9/16, 5/8]]
   1/2..17/32        PASS          [[3], [17/32, 9/16]]
 17/32..9/16         PASS          [[2], [9/16, 5/8]]
  9/16..5/8          fail          [[1], [5/8, 3/4]]
  9/16..19/32        PASS          [[2], [19/32, 5/8]]
 19/32..5/8          PASS          [[1], [5/8, 3/4]]
   5/8..3/4          fail          [3/4, 1]
   5/8..11/16        fail          [[1], [11/16, 3/4]]
   5/8..21/32        PASS          [[2], [21/32, 11/16]]
 21/32..11/16        PASS          [[1], [11/16, 3/4]]
 11/16..3/4          fail          [3/4, 1]
 11/16..23/32        PASS          [[1], [23/32, 3/4]]
 23/32..3/4          PASS          [3/4, 1]
   3/4..1            fail          EMPTY
   3/4..7/8          fail          [7/8, 1]
   3/4..13/16        fail          [[1], [13/16, 7/8]]
   3/4..25/32        PASS          [[2], [25/32, 13/16]]
 25/32..13/16        PASS          [[1], [13/16, 7/8]]
 13/16..7/8          fail          [7/8, 1]
 13/16..27/32        fail          [[1], [27/32, 7/8]]
 13/16..53/64        PASS          [[2], [53/64, 27/32]]
 53/64..27/32        PASS          [[1], [27/32, 7/8]]
 27/32..7/8          fail          [7/8, 1]
 27/32..55/64        PASS          [[1], [55/64, 7/8]]
 55/64..7/8          PASS          [7/8, 1]
   7/8..1            fail          EMPTY
   7/8..15/16        fail          [15/16, 1]
   7/8..29/32        fail          [[1], [29/32, 15/16]]
   7/8..57/64        PASS          [[2], [57/64, 29/32]]
 57/64..29/32        PASS          [[1], [29/32, 15/16]]
 29/32..15/16        fail          [15/16, 1]
 29/32..59/64        PASS          [[1], [59/64, 15/16]]
 59/64..15/16        PASS          [15/16, 1]
 15/16..1            fail          EMPTY
 15/16..31/32        fail          [31/32, 1]
 15/16..61/64        fail          [[1], [61/64, 31/32]]
 15/16..121/128       PASS          [[2], [121/128, 61/64]]
121/128..61/64        PASS          [[1], [61/64, 31/32]]
 61/64..31/32        fail          [31/32, 1]
 61/64..123/128       PASS          [[1], [123/128, 31/32]]
123/128..31/32        PASS          [31/32, 1]
 31/32..1            fail          EMPTY
```

```
  31/32..63/64          fail            [63/64, 1]
  31/32..125/128         PASS            [[1], [125/128, 63/64]]
 125/128..63/64          fail            [63/64, 1]
 125/128..251/256         PASS            [[1], [251/256, 63/64]]
 251/256..63/64          PASS            [63/64, 1]
  63/64..1              fail            EMPTY
  63/64..127/128         fail            [127/128, 1]
  63/64..253/256         PASS            [[1], [253/256, 127/128]]
 253/256..127/128         PASS            [127/128, 1]
 127/128..1              fail            EMPTY
 127/128..255/256         fail            [255/256, 1]
 127/128..509/512         PASS            [[1], [509/512, 255/256]]

 509/512..255/256         PASS            [255/256, 1]
 255/256..1              fail            EMPTY
 255/256..511/512         fail            [511/512, 1]
 255/256..1021/1024         PASS            [[1], [1021/1024,
511/512]]
 1021/1024..511/512         PASS            [511/512, 1]
 511/512..1              fail            EMPTY
 511/512..1023/1024         fail            [1023/1024, 1]
 511/512..2045/2048         PASS            [[1], [2045/2048,
1023/1024]]
 2045/2048..1023/1024         PASS            [1023/1024, 1]
 1023/1024..1              fail            EMPTY
 1023/1024..2047/2048         PASS            [2047/2048, 1]
 2047/2048..1              fail            EMPTY
 2047/2048..4095/4096         PASS            [4095/4096, 1]
 4095/4096..1              fail            EMPTY
 4095/4096..8191/8192         PASS            [8191/8192, 1]
 8191/8192..1              fail            EMPTY
 8191/8192..16383/16384          PASS            [16383/16384, 1]
 16383/16384..1              fail            EMPTY
 16383/16384..32767/32768          PASS            [32767/32768, 1]
 32767/32768..1              fail            EMPTY
 32767/32768..65535/65536          PASS            [65535/65536, 1]
 65535/65536..1              fail            EMPTY
 65535/65536..131071/131072         PASS            [131071/131072,
1]
 131071/131072..1              fail            EMPTY
 131071/131072..262143/262144         PASS
[262143/262144, 1]
 262143/262144..1              PASS            EMPTY
------------------------------------------------------------------
--------------------------
Number of Function Evaluations:    159
```

$$Student[Calculus1][SurfaceOfRevolution](y1, x = 0..a, output = plot, functionoptions = [color = white], scaling = constrained, lightmodel = light4)$$




Surface of revolution formed when

$f(x) = \frac{1}{10}\sqrt{5x\sqrt{280x+9} + 15x - 100x^2}$, $0 \le x \le 1$, is rotated about a horizontal axis.