KP Flowchart — Adjusted to Avoid Text Collision

Lump Solution
nu = 2(ln f)_{xx}, f rational

→ via long-wave limit, k → 0

Long-Wave Limit
k → 0: exponential → rational limit

3+1D KP Equation
(u_t + 6uu_x + u_{xxx})_x + λu_{yy} + αu_{zz} + βu_{xy} + γu_{xz} = 0

2+1D KP Equation
Set z = 0, α = β = γ = 0

→ (u_t + 6uu_x + u_{}xx})_x + λu_{yy} = 0

KP-II Equation
λ > 0: stable, weak transverse dispersion

→ (u_t + 6uu_x + u_{xxx})_x + λu_{yy} = 0

KP-I Equation
λ < 0: unstable, strong transverse dispersion

→ (u_t + 6uu_x + u_{xxx})_x - |λ|u_{yy} = 0

KdV Equation
u_t + 6uu_x + u_{xxx} = 0

→ 1D limit: u_y = 0, λ = 0

Burgers Equation
u_t + uu_x = 0

→ No dispersion: drop u_{xxx}

Hirota Bilinear Form
Use D-operators on τ-function

N-Soliton Structure
f = Σ exp(μ_in_i + Σ μ_iμ_j A_ij)

1-Soliton
f = 1 + e^{η}, η = kx + ly + rt + χ

2-Soliton
f = 1 + e^{η₁} + e^{η₂} + e^{η₁+η₂+A₁₂}

3-Soliton
Include triple terms + phases

Multi Soliton
General combinatorial structure

Core Equation: 3+1D KP (white)

PDE Reductions: to 2+1D, KdV, Burgers

Soliton Solutions: via Hirota bilinear method

Lump Solutions: via long-wave rational limit

 How to Use This Diagram:
• Begin with the 3+1D KP equation in the center.
• BLUE blocks show PDE reductions; boxed equations are derived forms.
• GREEN = Multi-soliton construction via Hirota's method.
• ORANGE = Long-wave limit produces rational (lump) solutions.
→ Each node contains a derived PDE or structure to explore symbolically or numerically.