Then Eqs. (9)–(11) take the form

$$(1+K)h'' + fh' - \frac{2n}{(n+1)}h^2 - \frac{2}{(n+1)}Mh + Kg' + \frac{2}{(n+1)}\sigma\theta = 0, \tag{18}$$

$$(1+\frac{K}{2})g'' + fg' - \left(\frac{3n-1}{n+1}\right)hg - \frac{2K}{(n+1)}\left(2g+h'\right) = 0, \tag{19}$$

$$\theta'' + \Pr f\theta' = 0, \tag{20}$$

and the corresponding boundary conditions now become

$$f = -\lambda,\;\; h = -1,\;\; g = -0.5h',\;\; \theta' = -c\left[1-\theta\left(0\right)\right] \;\text{ at }\; \eta = 0, \tag{21}$$

$$h \to 0,\;\; g \to 0,\;\; \theta \to 0 \;\text{ as }\; \eta \to \infty. \tag{22}$$

For the computational purposes, infinity has been fixed at 8. It is observed that the value greater than 8 does not result in any significant change in the numerical results.

3.1. Variational Formulation

The variational form associated with Eqs. (17)–(20) over a typical two-noded element (η_e, η_{e+1}) is given by

$$\int_{\eta_e}^{\eta_{e+1}} w_1 \left\{f' - h\right\} d\eta = 0, \tag{23}$$

$$\int_{\eta_e}^{\eta_{e+1}} w_2 \left\{(1+K)h'' + fh' - \frac{2n}{(n+1)}h^2 - \frac{2}{(n+1)}Mh + Kg' + \frac{2}{(n+1)}\sigma\theta\right\} d\eta = 0, \tag{24}$$

$$\int_{\eta_e}^{\eta_{e+1}} w_3 \left\{(1+\frac{K}{2})g'' + fg' - \left(\frac{3n-1}{n+1}\right)hg - \frac{2K}{(n+1)}\left(2g+h'\right)\right\} d\eta = 0, \tag{25}$$

$$\int_{\eta_e}^{\eta_{e+1}} w_4 \left\{\theta'' + \Pr f\theta'\right\} d\eta = 0, \tag{26}$$

where w_1, w_2, w_3, and w_4 are weight functions, which may be viewed as the variation in f, h, g, and θ, respectively.

3.2. Finite Element Formulation

The finite element model may be obtained from Eqs. (23)–(26) by substituting finite element approximation of the form

$$f=\sum_{j=1}^{2} f_j\psi_j,\quad h=\sum_{j=1}^{2} h_j\psi_j,\quad g=\sum_{j=1}^{2} g_j\psi_j,\quad \theta=\sum_{j=1}^{2} \theta_j\psi_j, \tag{27}$$

with $w_1=w_2=w_3=w_4=\psi_i\ (i=1,2)$, where the shape functions ψ_i for a typical line element (η_e,η_{e+1}) are given by:

$$\psi_1=\frac{\eta_{e+1}-\eta}{\eta_{e+1}-\eta_e},\quad \psi_2=\frac{\eta-\eta_e}{\eta_{e+1}-\eta_e},\quad \eta_e\le\eta\le\eta_{e+1}. \tag{28}$$

The finite element equations are, therefore, given by

$$\begin{bmatrix} [K^{11}][K^{12}][K^{13}][K^{14}] \\ [K^{21}][K^{22}][K^{23}][K^{24}] \\ [K^{31}][K^{32}][K^{33}][K^{34}] \\ [K^{41}][K^{42}][K^{43}][K^{44}] \end{bmatrix} \begin{bmatrix} \{f\} \\ \{h\} \\ \{g\} \\ \{\theta\} \end{bmatrix} = \begin{bmatrix} \{b^1\} \\ \{b^2\} \\ \{b^3\} \\ \{b^4\} \end{bmatrix}, \tag{29}$$

$[K^{mn}]$ and $[b^m]$ $(m,n=1,2,3,4)$ are the matrices of order 2×2 and 2×1, respectively, and are given by

$$K_{ij}^{11}=\int_{\eta_e}^{\eta_{e+1}}\psi_i\frac{d\psi_j}{d\eta}d\eta,\,K_{ij}^{12}=-\int_{\eta_e}^{\eta_{e+1}}\psi_i\psi_j d\eta,\;\; K_{ij}^{13}=K_{ij}^{14}=0,\quad K_{ij}^{21}=0,$$

$$K_{ij}^{22}=-(1+K)\int_{\eta_e}^{\eta_{e+1}}\frac{d\psi_i}{d\eta}\frac{d\psi_j}{d\eta}d\eta+\int_{\eta_e}^{\eta_{e+1}}\bar f\psi_i\frac{d\psi_j}{d\eta}d\eta-\frac{2n}{(n+1)}\int_{\eta_e}^{\eta_{e+1}}\bar h\psi_i\psi_j d\eta-\frac{2M}{(n+1)}\int_{\eta_e}^{\eta_{e+1}}\psi_i\psi_j d\eta,$$

$$K_{ij}^{23}=K\int_{\eta_e}^{\eta_{e+1}}\psi_i\frac{d\psi_j}{d\eta}d\eta,\;\; K_{ij}^{24}=\frac{2\sigma}{(n+1)}\int_{\eta_e}^{\eta_{e+1}}\psi_i\psi_j d\eta,\;\; K_{ij}^{31}=0,\;\; K_{ij}^{32}=-\frac{2K}{(n+1)}\int_{\eta_e}^{\eta_{e+1}}\psi_i\frac{d\psi_j}{d\eta}d\eta,$$

$$K_{ij}^{33}=-\left(1+\frac{K}{2}\right)\int_{\eta_e}^{\eta_{e+1}}\frac{d\psi_i}{d\eta}\frac{d\psi_j}{d\eta}d\eta+\int_{\eta_e}^{\eta_{e+1}}\bar f\psi_i\frac{d\psi_j}{d\eta}d\eta-\left(\frac{3n-1}{n+1}\right)\int_{\eta_e}^{\eta_{e+1}}\bar h\psi_i\psi_j d\eta$$

$$-\frac{4K}{(n+1)}\int_{\eta_e}^{\eta_{e+1}}\psi_i\psi_j d\eta,$$

$$K_{ij}^{34}=0,\;\; K_{ij}^{41}=K_{ij}^{42}=K_{ij}^{43}=0,\;\; K_{ij}^{44}=-\int_{\eta_e}^{\eta_{e+1}}\frac{d\psi_i}{d\eta}\frac{d\psi_j}{d\eta}d\eta+\Pr\int_{\eta_e}^{\eta_{e+1}}\bar f\psi_i\frac{d\psi_j}{d\eta}d\eta, \tag{30}$$

and