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FINITE ELEMENT ANALYSIS OF MHD EFFECTS ON MICROPOLAR FLUID FLOW IN A VERTICAL CHANNEL

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ABSTRACT

An analysis is presented for the problem of the fully developed natural convection under the influence of micropolar fluid flow of heat and mass transfer in a vertical channel. Asymmetric temperature and convection boundary conditions are applied to the walls of the channel. Solutions of the coupled non-linear governing equations are obtained for different values of the buoyancy ratio and various material parameters of the micropolar fluid and magnetic parameters. The resulting non dimensional boundary value problem is solved by the Galerken Finite element method using MATLAB Software. Influence of the governing parameters on the fluid flow as well as heat and solute transfers is demonstrated to be significant.

Keywords: -*Finite Element Method, MHD, Natural convection, Micropolarfluid, Vertical channel.*

INTRODUCTION

Problems dealing with free convection are often found in many industrial applications. Fully-developed free convection heat transfer in a vertical channel or parallel plates was considered by Bodia et al. [3] for symmetric walls temperature. Aung et al. [1, 2] and Miyatake et al. [7] studied the case of asymmetric boundary conditions which was later examined by Nelson et al. [8].

The earliest formulation of a general theory of fluid micricontinua was attributed to Eringen[5].His theory of micro fluids has opened up new areas in research in the physics of fluid flow. By Eringen's definition, a simple microfluid is a fluent medium whose properties and behaviour are affected by the local motions of the material particles contained in each of its volume elements such a fluid possesses local inertia. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium Lukaszewicz[6].

The basic idea of micropolar fluids has originated from the need to model the non-Newtonian flow of fluids containing rotating micro-constituents. Besides the usual equations for Newtonian flow, this theory introduces some new material parameters, an additional independent vector field-the microrotation-and new constitutive equations that must be solved simultaneously with the usual Newtonian flow equations. Subsequent studies showed that the model can be successfully applied to a wide range of applications including blood flow, lubricants, porous media, turbulent shear flows, and flow in capillaries, and microchannels

The free micropolar convection for the case of asymmetricboundary conditions or asymmetric heating was investigated by Chamkha et al. [4], Thin film flow of non-Newtonian fluidsona moving beltstudied A.M. Siddiquiet al.[9], HabibisSaleh et al.[10] studied the Simulation of Micropolar Fluid Flow in a vertical Channel usingHPM.As pointed out recently by Rawat and Bhargava [11], the study of heat and mass transfer in micropolarfluids is of importance in the fields of chemical engineering, aerospace engineering and also industrial manufacturingeffects processes.Sunil et al. [12] studied the Effect of Rotation on Double-Diffusive Convection in a Magnetized Ferrofluid with Internal Angular Momentum.

MATHEMATICAL MODEL

Consider the free convection of a micropolar fluid between two vertical plates, the space between the plates being h. The flow is assumed laminar, steady and fully-developed, i.e. the transverse velocity is zero. A uniform magnetic field is applied to the flow. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in an asymmetric heating situation.
The dimensionless governing equations are:

$$(1+K)\frac{d^2u}{dy^2}+K\frac{dN}{dy}+\theta = Mu \quad (1)$$

$$\left(1+\frac{K}{2}\right)\frac{d^2N}{dy^2}-K\left(2N+\frac{du}{dy}\right)=0 \quad (2)$$

$$\frac{d^2\theta}{dy^2}=0 \quad (3)$$

Subject to the boundary conditions

$u(0) = 0,\ \theta(0) = \mathrm{R},\ N(0) = 0,$ (4)

$u(1) = 0,\ \theta(1) = \mathrm{R}, N(1) = 0,$ (5)

where u is dimensionless velocity, N the dimensionless microrotation, θ dimensionless temperature, K is the material parameter and $R = (T_1 - T_o)/(T_2 - T_o)$, T_1 temperature of the left cooled wall and T_2 temperature of the right wall.

METHOD OF SOLUTION

The above model is a system of second-order BVP. Equation (1) subject to the boundary conditions (4) – (5), possesses the following Finite Element solution, obtained with the help of the MATLAB software. In order to reduce the above system of differential equations to a system of dimensionless form, we may represent the velocity and microrotation, temperature and concentration by applying the Galerkin finite element method for equation (1) over a typical two-noded linear element (e) $(y_j \leq y \leq y_k)$ is

$$u = N.\emptyset, N = [N_j\ , N_k],\ \ \emptyset = \begin{bmatrix} u_j \\ u_k \end{bmatrix}, N_j = \frac{y_k - y}{l}, \qquad N_k = \frac{y - y_j}{l}, l = y_k - y_j = h,$$

$$\int_{y_j}^{y_k} N^T \left[(1+K)\frac{d^2u}{dy^2} + K\frac{dN}{dy} + \theta - Mu\right] dy \qquad (6)$$

$$\int_{y_j}^{y_k} \left[(1+K)\frac{d^2u}{dy^2} + Mu - R\right] dy = 0 \quad where \quad R = K\frac{dN}{dy} + \theta$$

The element equation given by

$$\int_{y_j}^{y_k} (1+K)\begin{bmatrix} N_j' N_j' N_j' N_k' \\ N_k' N_j' N_k' N_k' \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + M\begin{bmatrix} \mathrm{N_j N_j} & \mathrm{N_j N_k} \\ \mathrm{N_k N_j} & \mathrm{N_k N_k} \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy - R\begin{bmatrix} N_j \\ N_k \end{bmatrix} dy = 0$$

$$\int_{y_j}^{y_k} S\, dy = \int_{y_j}^{y_k} R^* dy \qquad (7)$$

Where $S = (1+K)\begin{bmatrix} N_j' N_j' N_j' N_k' \\ N_k' N_j' N_k' N_k' \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + M\begin{bmatrix} \mathrm{N_j N_j} & \mathrm{N_j N_k} \\ \mathrm{N_k N_j} & \mathrm{N_k N_k} \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy\, and\, R^* = R\begin{bmatrix} N_j \\ N_k \end{bmatrix}$

$$S = \frac{(1+K)}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{Ml}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} and\, R^* = R\frac{l}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We write the element equation for the elements $y_{i-1} \leq y \leq y_i$ and $y_i \leq y \leq y_{i+1}$. Assembling these element equations, we get

$$\frac{(1+K)}{l}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{M\,l}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=R\frac{l}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \quad (8)$$

Now put row corresponding to the node i to zero, from equation (8) the difference schemes with $l = h$ is

$$\frac{(1+K)}{h}(-u_{i-1}+2u_i-u_{i+1})+\frac{Mh}{6}(u_{i-1}+4u_i+u_{i+1})=R^* \quad (9)$$

Also can written as

$$A_1u_{i-1}+A_2u_i+A_3u_{i+1}=A_4u_{i-1}+A_5u_i+A_6u_{i+1}+R^* \quad (10)$$

Similarly, the equations (2) and (3) are becoming as follows:

$$B_1N_{i-1}+B_2N_i+B_3N_{i+1}=B_4N_{i-1}+B_5N_i+B_6N_{i+1}+R_1^* \quad (11)$$

$$C_1T_{i-1}+C_2T_i+C_3T_{i+1}=C_4T_{i-1}+C_5T_i+C_6T_{i+1} \quad (12)$$

$$A_1=\left(\frac{\mathrm{Mh}}{6}-\left(\frac{1+K}{h}\right)\right), A_2=\left(2\left(\frac{1+K}{h}\right)+\frac{\mathrm{Mh}}{3}\right), A_3=\left(\frac{\mathrm{Mh}}{6}-\left(\frac{1+K}{h}\right)\right),$$

$$A_4=\left(\left(\frac{1+K}{h}\right)-\frac{\mathrm{Mh}}{6}\right), A_5=,-\left(2\left(\frac{1+K}{h}\right)+\frac{\mathrm{Mh}}{3}\right), A_6=\left(\left(\frac{1+K}{h}\right)-\frac{\mathrm{Mh}}{6}\right),$$

$$B_1=\left(\frac{hBK}{3}-\frac{1+\frac{K}{2}}{h}\right), B_2=\left(\frac{4hBK}{3}+2\left(\frac{1+\frac{K}{2}}{h}\right)\right), B_3=\left(\frac{hBK}{3}-\frac{1+\frac{K}{2}}{h}\right),$$

$$B_4=\left(\frac{1+\frac{K}{2}}{h}-\frac{hBK}{3}\right), B_5=-\left(\frac{4hBK}{3}+2\left(\frac{1+\frac{K}{2}}{h}\right)\right), B_6=\left(\frac{1+\frac{K}{2}}{h}-\frac{hBK}{3}\right),$$

$$C_1=-\frac{1}{h},\ C_2=\frac{2}{h}, C_3=-\frac{1}{h}, C_4=\frac{1}{h},\ C_5=-\frac{2}{h}, C_6=\frac{1}{h},$$

$$R^*=h\left(K\left(\frac{N_{i+1}-N_{i-1}}{h}\right)+\theta\right), R_1^*=-h\left(\frac{u_{i+1}-u_{i-1}}{h}\right),$$

In equation (2)-(3), taking i=1(1) n and using boundary conditions (4) and (5), the following system of equation are obtained.

$$A_iX_i=B_i\,,\ i=1,2,3\,.... \quad (13)$$

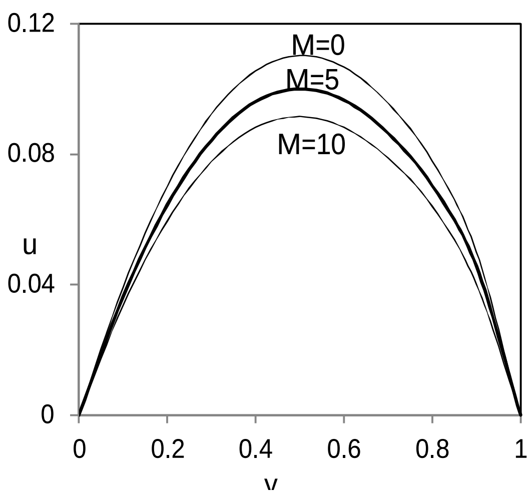
Where A_i's are matrices of order n and X_i and B_i's are column matrices having n-components. The solution of above system of equations are obtained using Thomas algorithm for velocity, angular velocity and temperature. Also, numerical solutions for these equations are obtained by MATLAB program. In order to prove the convergence and stability of the method, the same MATLAB-program was run with slightly changed values of h and k, no significant change was observed in the values of u, N, θ.

RESULTS AND DISCUSSION

To get the physical insight of the problem the results are discussed through graph. The computations are carried out for the governing flow of the problem with the Galerkin Finite Element Method. The obtained results are compared and these are a good agreement with the result of Habibis Saleh at al [10]. The numerical results are obtained for the velocity; microrotation and temperature have been shown graphically for different flow parameters.

The effect of magnetic field parameter M on the velocity profiles u and mocrorotation N for K=5,when R=0.5 is shown in Fig.1. Here it is observed that the velocity profiles decreases with an increase of M, microrotation profiles increases up to centre of the channel the reverse phenomenon is observed in the other part of the channel. It can be seen that the velocity profiles u increases with an increase of M, The microrotation N decreases with the increase of M, up to middle of the channel (flow direction is upward) and it is increases with increase of M, in the other part of the channel is observed.

The Fig.2 illustrates the influence of vortex viscosity parameter K on the distribution of velocity u and microrotation N for R=0.5. It is observed that with the increasing the value of K the intensity of convective velocity u is reduced as compared to the Newtonian fluid situation (K=0). In fact it is found that as K→ ∞, u→ 0.The influence of parameter K on the microrotation N it is noticed that the variation with K of the value of N evaluated at the position half of the channel also presented in the graphs it can be seen that the intensity of N first increases with increase of K, the reverse phenomenon is observed later. The velocity and microrotation profiles for the case R = 0.5 and K= 0, 1, 3 are depicted in Fig.2 Clearly; good agreement is observed when we compared with HabibisSaleh at al [10].



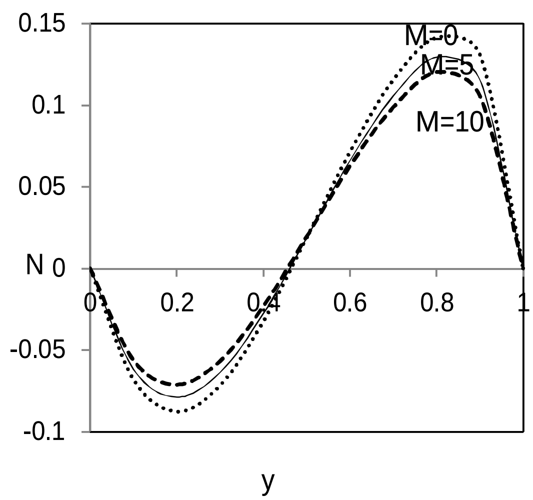


Figure1: Effects ofMagnetic parameter***M***onthe velocity profiles ***u*** and the microrotation ***N*** for ***K=5***and ***R=0.5.***

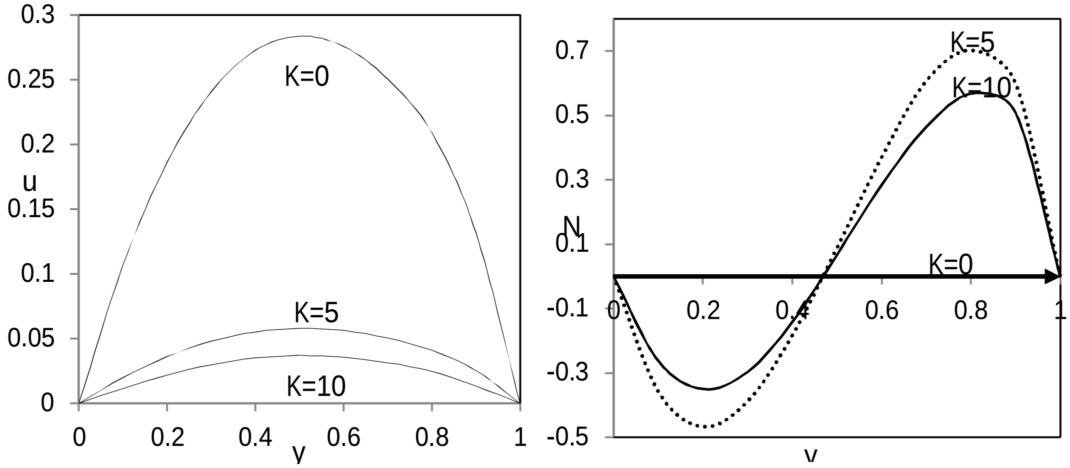


Figure 2: The velocity profiles and microrotation for ***R=0.5***,

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