

$$Sc = \frac{HV_0}{D}. \quad (8)$$

The governing equations now reduce to the following pair of non-linear partial differential equations in terms of dimensionless longitudinal velocity, U and dimensionless species concentration parameter, Φ , viz:

$$\begin{aligned} \frac{\partial U}{\partial T} + \frac{\partial U}{\partial Y} + \frac{\partial P}{\partial x} \\ = \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} - NmU - \frac{1}{\lambda} U - N_f U^2, \end{aligned} \quad (9)$$

$$\frac{\partial \Phi}{\partial T} + \frac{\partial \Phi}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 \Phi}{\partial Y^2}. \quad (10)$$

The corresponding transformed boundary conditions are now given by:

$$U = 0, \quad \Phi = 1 \quad \text{at } Y = 0 \text{ (lower channel wall),} \quad (11)$$

$$U = 0, \quad \Phi = 1 \quad \text{at } Y = 1 \text{ (upper channel wall).} \quad (12)$$

Equations (9) and (10) have been solved using the finite element method. To simplify a numerical solution, the pressure gradient in (9) is decomposed into a steady component and an imposed (oscillatory) component as follows:

$$-\frac{\partial P}{\partial X} = \left[\frac{\partial P}{\partial X} \right]_s + \left[\frac{\partial P}{\partial X} \right]_o e^{i\omega t}. \quad (13)$$

Many studies have appeared using this approach. For example, Mazumder and Das examined the pulsatile mass transfer of a contaminant in a conduit [30] using this approach. Their study however does not consider magnetic field or porous effects. In the present study, we shall examine the interaction of pulsation pressure and species diffusion and also the influence of porous media drag forces, magnetic body force, on biofluid flow velocity profile in a channel. We have also studied the variation of maximum velocity in the channel with time.

4 Numerical solution

The governing equations amount to a pair of non-linear and linear partial differential equations with

corresponding boundary conditions. Finite element solutions have been obtained for the present two-point boundary value problem. An excellent description of the method is available in Ref. [31]. Herein we shall briefly describe the numerical technique. In the computations, pressure gradient is re-defined as:

$$-\frac{\partial P^*}{\partial X} = Ps + Po(\cos \omega^* T). \quad (14)$$

Using (14), the momentum equation (9) takes the form:

$$\begin{aligned} \frac{\partial U}{\partial T} + \frac{\partial U}{\partial Y} - Ps - Po(\cos \omega^* T) \\ = \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} - NmU - \frac{1}{\lambda} U - N_f U^2. \end{aligned} \quad (15)$$

The corresponding boundary conditions are:

$$U = 0, \quad \Phi = 1 \quad \text{at } Y = 0 \text{ (lower channel wall),} \quad (16)$$

$$U = 0, \quad \Phi = 0 \quad \text{at } Y = 1 \text{ (upper channel wall).} \quad (17)$$

The initial conditions in time, are:

$$U = 0, \quad \Phi = 1 \quad \text{at } T = 0. \quad (18)$$

For numerical robustness $Y = \infty$ has been fixed as 1. The whole domain is divided into a set of 100 line elements of equal width, each element being two-noded.

4.1 Variational formulation

The variational form associated with (15) and (10) over a typical two-noded linear element (Y_e, Y_{e+1}) is given by

$$\begin{aligned} \int_{Y_e}^{Y_{e+1}} w_1 \left\{ -\frac{\partial U}{\partial T} - \frac{\partial U}{\partial Y} + Ps + Po(\cos \omega^* T) \right. \\ \left. + \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} - \frac{1}{\lambda} U - NmU - N_f U^2 \right\} dY \\ = 0, \end{aligned} \quad (19a)$$

$$\int_{Y_e}^{Y_{e+1}} w_2 \left\{ \frac{1}{Sc} \frac{\partial^2 \Phi}{\partial Y^2} - \frac{\partial \Phi}{\partial Y} - \frac{\partial \Phi}{\partial T} \right\} dY = 0, \quad (19b)$$

where w_1 and w_2 are arbitrary test functions and may be viewed as the variation in U and Φ , respectively.

4.2 Finite element formulation

The finite element model may be obtained from (19a) to (19b) by substituting finite element approximations of the form:

$$U = \sum_{j=1}^2 U_j \psi_j, \quad \Phi = \sum_{j=1}^2 \Phi_j \psi_j, \quad (20)$$

where $w_1 = w_2 = \psi_i (i = 1, 2)$ and ψ_i are the shape functions for a typical element (Y_e, Y_{e+1}) and are taken as:

$$\begin{aligned} \psi_1^{(e)} &= \frac{Y_{e+1} - Y}{Y_{e+1} - Y_e}, \\ \psi_2^{(e)} &= \frac{Y - Y_e}{Y_{e+1} - Y_e}, \quad Y_e \leq Y \leq Y_{e+1}. \end{aligned} \quad (21)$$

The finite element model of the equations thus formed is given by:

$$[KU_{ij}] [\{U_i\}] + [MU_{ij}] [\{U'_i\}] = [\{F_{U_i}\}] \quad (22)$$

$$\text{and } [K\Phi_{ij}] [\{\Phi_i\}] + [M\Phi_{ij}] [\{\Phi'_i\}] = [\{F_{\Phi_i}\}], \quad (23)$$

where $([KU_{ij}], [K\Phi_{ij}])$ and $([\{F_{U_i}\}], [\{F_{\Phi_i}\}])$ ($i, j = 1, 2$) are the matrices of order 2×2 , and 2×1 , respectively. Also U'_i and Φ'_i are derivatives of U_i and Φ_i with respect to T . All these matrices may be defined as follows:

$$\begin{aligned} KU_{ij} &= -\frac{1}{\text{Re}} \int_{Y_e}^{Y_{e+1}} -\frac{d\psi_i}{dY} \frac{d\psi_j}{dY} dY - \int_{Y_e}^{Y_{e+1}} \psi_i \frac{d\psi_j}{dY} dY \\ &\quad - Nm \int_{Y_e}^{Y_{e+1}} \psi_i \psi_j dY - \frac{1}{\lambda} \int_{Y_e}^{Y_{e+1}} \psi_i \psi_j dY \\ &\quad - Nf \bar{U}_1 \int_{Y_e}^{Y_{e+1}} \psi_i \psi_1 \psi_j dY \\ &\quad - Nf \bar{U}_2 \int_{Y_e}^{Y_{e+1}} \psi_i \psi_2 \psi_j dY, \end{aligned} \quad (24)$$

$$MU_{ij} = - \int_{Y_e}^{Y_{e+1}} \psi_i \psi_j dY, \quad (25)$$

$$K\Phi_{ij} = -\frac{1}{Sc} \int_{Y_e}^{Y_{e+1}} \frac{d\psi_i}{dY} \frac{d\psi_j}{dY} dY - \int_{Y_e}^{Y_{e+1}} \psi_i \frac{d\psi_j}{dY} dY, \quad (26)$$

$$M\Phi_{ij} = - \int_{Y_e}^{Y_{e+1}} \psi_i \psi_j dY, \quad (27)$$

$$\begin{aligned} F_{U_i} &= -\frac{1}{\text{Re}} \left(\psi_i \frac{dU}{dY} \right)_{Y_e}^{Y_{e+1}} \\ &\quad - \int_{Y_e}^{Y_{e+1}} \psi_i (Ps + Po(\cos\omega * T)) dY, \end{aligned} \quad (28)$$

$$F_{\Phi_i} = -\frac{1}{Sc} \left(\psi_i \frac{d\Phi}{dY} \right)_{Y_e}^{Y_{e+1}}, \quad (29)$$

where

$$\bar{U} = \sum_{i=1}^2 \bar{U}_i \psi_i, \quad \bar{\Phi} = \sum_{i=1}^2 \bar{\Phi}_i \psi_i. \quad (30)$$

Each element matrix is of the order 2×2 . The entire flow domain is discretized into a set of 100 line elements. Post-assembly of all the element equations leads to a matrix of order 101×101 . The system of equations generated after assembly of the element equations is *non-linear* and an iterative scheme is employed for a robust solution. The system is linearized by incorporating the functions $\bar{U}$ and $\bar{\Phi}$, which are assumed to be known. After applying the given initial and boundary conditions a reduced system of 99 equations remains which is solved using the Gauss elimination method. In the present paper, we have computed the velocity and mass transfer distributions in the channel over space and time. The present analysis serves to provide in particular a systematic examination of the interactive effects of the problem control parameters, i.e., Darcian parameter (λ), Forchheimer parameter (Nf), Schmidt number (Sc), Reynolds number (Re), hydromagnetic number (Nm), non-dimensional time (T) and pulsatile coefficients (Ps, Po) on the flow regime.