"Is there a convert command that converts a matrix to a tensor"

$with(Physics):$

First of all, even working with matrices, you are working in some *space* and, in general, using some coordinates, and if working with tensors you use certain type of letters as *tensor indices*. Suppose it is a 3D Euclidean space, your coordinates are Cartesian, and you want to use lowercase letters as indices; set that:

$Setup(dimension = 3, metric = euclidean, coordinates = cartesian, spacetimeindices = lowercase)$

The dimension and signature of the tensor space are set to $[3, (+ + +)]$

Systems of spacetime coordinates are: $\{X = (x, y, z)\}$

The Euclidean metric in coordinates $[x, y, z]$

$$g_{\mu, \nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[coordinatesystems = \{X\}, dimension = 3, metric = \{(1, 1) = 1, (2, 2) = 1, (3, 3) = 1\}, spacetimeindices = lowercaselatin] \quad (1)$$

Suppose you have the matrix

$Matrix(3, symbol = m)$

$$\begin{bmatrix} m_{1, 1} & m_{1, 2} & m_{1, 3} \\ m_{2, 1} & m_{2, 2} & m_{2, 3} \\ m_{3, 1} & m_{3, 2} & m_{3, 3} \end{bmatrix} \quad (2)$$

To transform it into a tensor, just use the Define command

$Define(M[i, j] = \mathbf{(2)})$

Defined objects with tensor properties

$$\{\gamma_a, M_{i, j}, \sigma_a, \partial_a, g_{a, b}, \epsilon_{a, b, c}, X_a\} \quad (3)$$

That is all. You can perform all tensor computations with such so defined tensor $M_{i, j}$. Start with its definition

$M[definition]$

$$M_{i,j} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{4}$$

Then itscomponents (default is the *covariant* ones)

$M[\ ]$

$$M_{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{5}$$

The *contravariant* components are the same as the covariant ones because the space is Euclidean

$M[\sim]$

$$M^{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{6}$$

The trace

$M[trace]$

$$m_{1,1} + m_{2,2} + m_{3,3} \tag{7}$$

Also via

$M[j,j]$

$$M_{j,j} \tag{8}$$

$SumOverRepeatedIndices(\mathbf{(8)})$

$$m_{1,1} + m_{2,2} + m_{3,3} \tag{9}$$

The determinant

$M[determinant]$

$$m_{1,1} m_{2,2} m_{3,3} - m_{1,1} m_{2,3} m_{3,2} - m_{1,2} m_{2,1} m_{3,3} + m_{1,2} m_{2,3} m_{3,1} + m_{1,3} m_{2,1} m_{3,2} - m_{1,3} m_{2,2} m_{3,1} \tag{10}$$

The inert representation of this determinant

$\%M[determinant]$

$$|\mathbf{M}| \tag{11}$$

$value(\mathbf{(11)})$

$$m_{1,1} m_{2,2} m_{3,3} - m_{1,1} m_{2,3} m_{3,2} - m_{1,2} m_{2,1} m_{3,3} + m_{1,2} m_{2,3} m_{3,1} + m_{1,3} m_{2,1} m_{3,2} - m_{1,3} m_{2,2} m_{3,1} \tag{12}$$

The non-zero components

$M[nonzero]$

$$M_{a,b} = \{(1,1) = m_{1,1}, (1,2) = m_{1,2}, (1,3) = m_{1,3}, (2,1) = m_{2,1}, (2,2) = m_{2,2}, (2,3) = m_{2,3}, (3,1) = m_{3,1}, (3,2) = m_{3,2}, (3,3) = m_{3,3}\} \tag{13}$$

*"for example to perform basic operations like omega*J*omega where J is the inertia tensor and omega the vector of angular velocity?"*

OK. Define your *vector* now. You can do that directly with Define, or also starting with a Maple *Vector* constructor

$Vector(3, symbol = v)$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{14}$$

$Define(V[j] = \mathbf{(14)})$

Defined objects with tensor properties

$$\{\gamma_a, M_{i,j}, \sigma_a, V_j, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \tag{15}$$

For example, the computation you asked for

$V[i]\, M[i,j]\, V[j]$

$$V_i M_{i,j} V_j \tag{16}$$

$SumOverRepeatedIndices(\mathbf{(16)})$

$$m_{1,1} v_1^2 + m_{1,2} v_1 v_2 + m_{1,3} v_1 v_3 + m_{2,1} v_1 v_2 + m_{2,2} v_2^2 + m_{2,3} v_2 v_3 + m_{3,1} v_1 v_3 + m_{3,2} v_2 v_3 + m_{3,3} v_3^2 \tag{17}$$

Another example: operations with the metric

$V[a]\, g\_[a,i]\, M[i,j]\, g\_[j,b] V[b]$

$$V_a g_{a,i} M_{i,j} g_{b,j} V_b \tag{18}$$

$Simplify(\mathbf{(18)})$

$$V_i M_{i,j} V_j \tag{19}$$

Or products of tensor (underlying matrices) that take the symmetry properties into account

$LeviCivita[a,b,c]\, V[a]\, V[b]$

$$\epsilon_{a,b,c} V_a V_b \tag{20}$$

$Simplify(\mathbf{(20)})$

$$0 \tag{21}$$

Or other tensor commands

$Symmetrize(M[a,b])$