Rational Trigonometry

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RationalTrigonometry[LnPeqns] Makes line and plane equations from points and vectors.

Calling Sequence

1) **LnPeqns**(**p1 , p2 , vars=[x,y], opt**) Line from two 2D points. Must be lists.

2) **LnPeqns**(**p , v , vars=[x,y], opt**) Line from 2D points and direction vector. Point is a list Direction, column vector v= <a,b>.

3) **LnPeqns**(**v**, **vars=[x,y], opt**) Line from vector column. **v=<a,b,c>**

4) **LnPeqns** **p1 , p2 , p3 , { colour:= GeomClr }, {dir=perpendicular**} ,vars=[x,y], **opt**) Line from two 2D points. Perpendicular or parallel line through third point specified by dir. Intersection point also returned. Must be lists . Applies to Blue, Red and Green geometry.

5) **LnPeqns**(**p1 , v , p2 , { colour:= GeomClr }, {dir=perpendicular} ,vars=[x,y], opt**) Line from a 2D point plus direction vector (column). Perpendicular or parallel line through point specified by dir. Intersection point also returned. Points are lists. Applies to Blue, Red and Green geometry.

6) **LnPeqns**(**l , p , { colour:= GeomClr } , {dir=NULL} ,vars=[x,y] , opt**) Line equation plus point tocheck for coincidence or create perpendicular or parallel line. Intersection point also returned. Applies to Blue, Red and Green geometry.

7) **LnPeqns**(**p3d ,v3d {point:=NULL} , {varp=lambda } , opt**) Createls 3D line parametric vector from a list and direction vector. If optional point given will also return the perpendicular line and intersection point.

8) **LnPeqns**(**[p3d ,v3d] {point:=NULL} , {varp=lambda } , opt**) Plane for a 3D point an vevtor normal to the plane. Optional point to give perpendicular line thro' point to plane and intersection point.

9) **LnPeqns**(**[p1 , p2 , p3] , {point:=NULL} , { vars:=[x,y,z] } ,{varp:=lambda}, opt**) Creates a plane from list of 3 points. Optional 4th point to give perpendicular line thro' point to plane and intersection point.

10) **LnPeqns**(**[p1 , v1 , v2] , {point:=NULL} , { vars:=[x,y,z] } ,{varp:=lambda} , opt**) Creates a plane from a point plus two direction vectors. Optional point to give perpendicular line thro' point to plane and intersection point.

11) **LnPeqns**(**l1 , l2 , opt**) **l1** and **l2** are 2D line eqations. Reports if lines are parallel or perpendicular in any of the Blue, Red or Green geometries.

Parameters

1) **p0** and **p1** - Represents a pair of 2D points. Type list.

2) **p , v** - Represents a 2D point. Defined as above. **v** is a column vector **<x , y >**

3) **v** - Represents a column vector of the line coefficients.**<a , b , c>**.

4) **p1**, **p2** and **p3** - Represents three 2D points. Defined as above.

5) **p1 , v , p2** - Represents a 2D point, column vector and 2D point. Defined as above

6) **l** and **p** - Represents a line equation and 2D point.

7) **p3d,v3d** and **point** - Represents a 3D line and plane The 3D line can be represented in 4 alternative formmats **[[x1 , y1 , z1] , < xv , yv , zv>]** , **[[x1, y1 , z1] , [x2, y2, z2]]**

or **[[x1, y1 , z1] , [[x2, y2, z2] , [x3, y3, z3]]]** i.e. a point and 2 points to define the direction vector.

Or $[\langle x1+\lambda\cdot xv, y1+\lambda\cdot yv, z1+\lambda\cdot zv\rangle]$. i.e. a parametric vector for the 3D line.

A plane, can be a list or column vector **[a , b , c , d]** or **< a , b , c , d >**. An equation $a\cdot x+b\cdot y+c\cdot z+d$ or a function $f(x,y,z)\to a\cdot x+b\cdot y+c\cdot z+d$ and either may contain an `=`.

8) **[p3d ,v3d]**

9) **[p1 , p2 , p3]** - the list of 3 three points define a plane. the 4th point ca

10) **[p1 , v1 , v2]** - a point plus two direction vectors to define a plane

11) **l1 , l2** - are in the format a $a\cdot x+b\cdot y+c$

Options

- **colour** type:- string can be **"Blue"**, **"Red"** or **"Green"**. Variations on spelling are permitted e.g. "b","B" and "blue". The default value **"Blue"** is set by the global variable **Geomclr** which can be thanged.
- **dir** default value is **NULL**, can be set to can be set to **parallel** or **perendicular**
- **vars** type:- list for two or three variables.The defaults are **[x , y]** or **[x , y , z]**.

- **varp** type:- name for a parametric variable. The default is **lambda**
- **opt** - **true** or **false** to display message. The default value is **true** set by the global variable **Prntmsg** which can be changed.

Description

- Note, do not use this command with projective point or line vectors. See **LPproj** instead.
- This command is part of the **RationalTrigonpmetry** package, so it can be used in the form **LnPeqns(..)** only after executing the command `with(RationalTrigonometry)`. However, it can always be accessed through the long form of the command by using **RationalTrigonometry:-LnPeqns**`(..)`.

Examples

```
> restart:with(RationalTrigonometry):
```

```
> #1  2 points
  LnPeqns([3,7],[5,2]);
  LnPeqns([3,7],[5,2],vars=[r,s],false);
```

$$-5\,x-2\,y+29$$
$$-5\,r-2\,s+29 \qquad (4.1)$$

```
>   #2; point and direction vector
  LnPeqns([3,7],<6,5>,vars=[r,s]);
```

$$5\,r+27-6\,s \qquad (4.2)$$

```
> #3 2D Line from column vector Note the 2 entered after the vector to incidate 2D
  LnPeqns(<3,-7,8>,2);  #No message displayed
  LnPeqns(<3,-7,8>,2,vars=[r,s]);
```

$$3\,x-7\,y+8$$
$$3\,r-7\,s+8 \qquad (4.3)$$

```
> #4 Line from 2 points,  parallel and three types of perpendiculars
  LnPeqns([3,7],[5,2],point=[17,-3],dir=parallel);
  LnPeqns([3,7],[5,2],point=[17,-3]);
  LnPeqns([3,7],[5,2],point=[17,-3],colour="red");
  LnPeqns([3,7],[5,2],point=[17,-3],colour="green");
```

"Line thro' 1st two points and parallel thro' 3rd point"

$$-5\,x-2\,y+29,\ -5\,x+79-2\,y$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$-5\,x-2\,y+29,\ 2\,x-49-5\,y,\ \left[\frac{243}{29},\ -\frac{187}{29}\right]$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$-5\,x-2\,y+29,\ 2\,x-19+5\,y,\ \left[\frac{107}{21},\ \frac{37}{21}\right]$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$-5\,x-2\,y+29,\ 5\,x-91-2\,y,\ \left[12,\ -\frac{31}{2}\right] \qquad (4.4)$$

```
> #5 Line from point and direction vector,  parallel and three types of perpendiculars
  LnPeqns([3,7],<-1,2>,point=[5,2],dir=parallel);
  LnPeqns([3,7],<-1,2>,point=[5,2]);
  LnPeqns([3,7],<-1,2>,point=[5,2],colour="red");
  LnPeqns([3,7],<-1,2>,point=[5,2],colour="green");
```

"Line thro' 1st two points and parallel thro' 3rd point"

$$2\,x-13+y,\ 2\,x-12+y$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$2\,x-13+y,\ -x+1+2\,y,\ \left[\frac{27}{5},\ \frac{11}{5}\right]$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$2x - 13 + y,\ -x + 9 - 2y,\ \left[\frac{17}{3}, \frac{5}{3}\right]$$

"Line thro' 1st two points , perpendicular thro' 3rd point ans intersect point"

$$2x - 13 + y,\ -2x + 8 + y,\ \left[\frac{21}{4}, \frac{5}{2}\right] \qquad \textbf{(4.5)}$$

```
> #6 Line equation and point. Various input formats for line equation
  LnPeqns(2*x-5*y=-6,point=[3,12/5]);   #This checks if line and point are coincident
  LnPeqns(2*x-5*y-6,point=[3,4],dir=`parallel`);
  LnPeqns(2*x-5*y-6,point=[3,4],dir=`perpendicular`);
  f:=(x,y)->2*x-5*y-6:
  LnPeqns(f(x,y),point=[3,4],colour="red",dir=`perpendicular`);
  LnPeqns(S=2/5*R-6/5,point=[3,4],vars=[R,S],colour="green",dir=`perpendicular`);
```

"If = 0 line and point are coincident"

0

"Parallel line through given point"

$$2x + 14 - 5y$$

"Blue Prependicular to line through given point and intersect point "

$$5x - 23 + 2y,\ \left[\frac{127}{29}, \frac{16}{29}\right]$$

"Red Prependicular to line through given point and intersect point "

$$5x - 7 - 2y,\ \left[\frac{23}{21}, -\frac{16}{21}\right]$$

"Green Prependicular to line through given point and intersect point "

$$2R - 26 + 5S,\ [8, 2] \qquad \textbf{(4.6)}$$

```
> #7 3D line and optional perpendicular.
  LnPeqns([-4,1,3],<8,7,2>);
  LnPeqns([-4,1,3],<8,7,2>,point=[1,9,-7]);
  LnPeqns([-2,-5,3],<1,-1,2>,varp=alpha,point=[1,1,-1]);
  LnPeqns([-4,1,3],[4,5,1]); #2 3D points;
  # or
  LnPeqns([[-4,1,3],[4,5,1]]); #2 3D points as a listlist;
  LnPeqns(<-4+8*alpha,1+7*alpha,3+2*alpha>,point=[1,9,-7],varp=[alpha, beta])
```

"3D line equation"

$$\begin{bmatrix} -4 + 8\alpha \\ 1 + 7\alpha \\ 3 + 2\alpha \end{bmatrix}$$

"3D line equation, perpendicluar line through the point and intersection point "

$$\begin{bmatrix} -4 + 8\alpha \\ 1 + 7\alpha \\ 3 + 2\alpha \end{bmatrix}, \begin{bmatrix} 1 + \frac{23}{117}\beta \\ 9 - \frac{404}{117}\beta \\ -7 + \frac{1322}{117}\beta \end{bmatrix}, \left[\frac{140}{117}, \frac{649}{117}, \frac{503}{117}\right]$$

"3D line equation, perpendicluar line through the point and intersection point "

$$\left[\begin{array}{c} -2+\alpha_1 \\ -5-\alpha_1 \\ 3+2\,\alpha_1 \end{array}\right], \left[\begin{array}{c} 1-\frac{29}{6}\,\alpha_2 \\ 1-\frac{25}{6}\,\alpha_2 \\ -1+\frac{1}{3}\,\alpha_2 \end{array}\right], \left[-\frac{23}{6}, -\frac{19}{6}, -\frac{2}{3}\right]$$

"3D line equation"

$$\left[\begin{array}{c} -4+8\,\alpha \\ 1+4\,\alpha \\ 3-2\,\alpha \end{array}\right]$$

"3D line equation"

$$\left[\begin{array}{c} -4+8\,\alpha \\ 1+4\,\alpha \\ 3-2\,\alpha \end{array}\right]$$

"3D line equation, perpendicluar line through the point and intersection point "

$$\left[\begin{array}{c} -4+8\,\alpha \\ 1+7\,\alpha \\ 3+2\,\alpha \end{array}\right], \left[\begin{array}{c} 1+\frac{23}{117}\,\beta \\ 9-\frac{404}{117}\,\beta \\ -7+\frac{1322}{117}\,\beta \end{array}\right], \left[\frac{140}{117}, \frac{649}{117}, \frac{503}{117}\right] \qquad \textbf{(4.7)}$$

```
> #7 Plane and optional perpendicular.
  LnPeqns([[-4,1,3],<8,7,2>]);
  LnPeqns([[-2,-5,3],<1,-1,2>],point=[1,1,-1]);
```

"Plane from point & normal vector "

$$8\,x+7\,y+2\,z+19$$

"Plane from point & normal vector. Perpendicular line through 'point' and intersection point "

$$x-y+2\,z-9, \left[\begin{array}{c} 1+\alpha \\ 1-\alpha \\ -1+2\,\alpha \end{array}\right], \left[\frac{17}{6}, -\frac{5}{6}, \frac{8}{3}\right] \qquad \textbf{(4.8)}$$

```
> #8 & 9 Plane. Note list input for plane points or point & 2 vectors The vectors lie in the plane
  LnPeqns([[1,2,3],[-2,3,7],[8,6,2]]);
  LnPeqns([[1,2,3],[-2,3,7],[8,6,2]],point=[3,0,2]);
  LnPeqns([[0,4,1],<-2,3,7>,<8,6,2>],varp=kappa,point=[1,9,1]);
```

"Plane from 3 points"

$$-17\,x+25\,y-19\,z+24$$

"Plane from 3 points, perpendicular line through point and intersection point "

$$-17\,x+25\,y-19\,z+24, \left[\begin{array}{c} 3-17\,\alpha \\ 25\,\alpha \\ 2-19\,\alpha \end{array}\right], \left[\frac{32}{15}, \frac{65}{51}, \frac{263}{255}\right]$$

"Plane from point & 2 vectors, perpendicular line through the point and intersection point "

$$-3x+5y-3z-17,\begin{bmatrix}1-36\kappa\\ 9+60\kappa\\ 1-36\kappa\end{bmatrix},\left[\frac{109}{43},\frac{277}{43},\frac{109}{43}\right] \tag{4.9}$$

```
> #10 Information about two line equations
  LnPeqns(  2*x-5*y-6,5*x-1/13*y+2);
  LnPeqns(  2*x-13+y , 2*x-12+y );
  LnPeqns(-x + 1 + 2*y, 2*x - 13 + y);
  LnPeqns( 2*x - 13 + y, -x + 9 - 2*y);
  LnPeqns( S = 2/5*R-6/5 ,(2*R)/5 - 26/5 + S,vars=[R,S])
```

"Lines intersect at "

$$\left[-\frac{8}{19},-\frac{26}{19}\right]$$

"Lines are parallel "

"Lines are Blue perpendicular intersection point is "

$$\left[\frac{27}{5},\frac{11}{5}\right]$$

"Lines are Red perpendicular intersection point is "

$$\left[\frac{17}{3},\frac{5}{3}\right]$$

"Lines are Green perpendicular intersection point is "

$$[8,2] \tag{4.10}$$

```
> #11 convert line expression to vector format;
  LnPeqns(y=5*x-3);  #No message displayed
  LnPeqns~([y=5*x-3,(x,y)->x+2*y=4,x-2]);
```

"2D Line in Vector format"

$$\begin{bmatrix}-5\\ 1\\ 3\end{bmatrix}$$

"2D Line in Vector format"

"2D Line in Vector format"

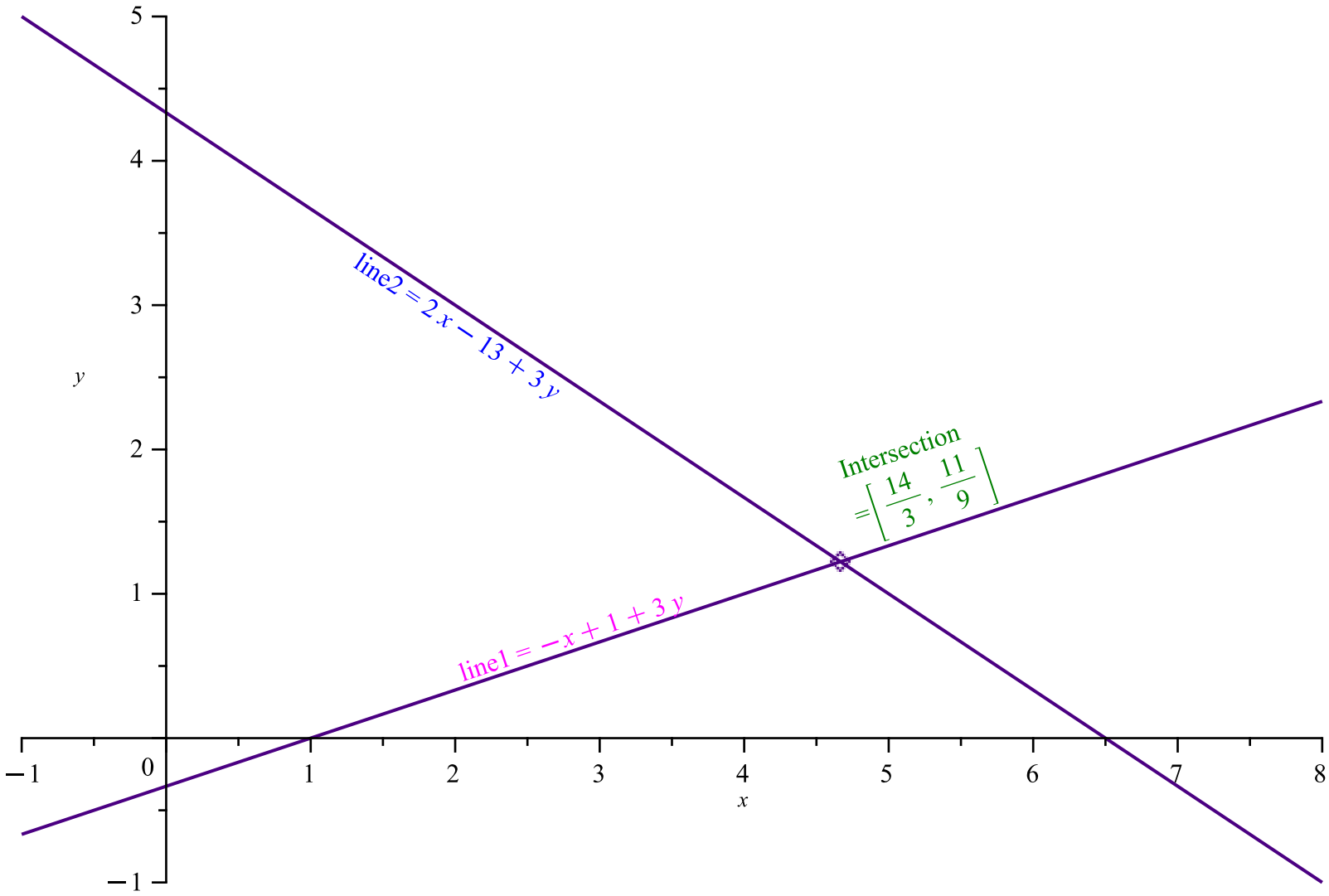
"2D Line in Vector format"

$$\left[\begin{bmatrix}-5\\ 1\\ 3\end{bmatrix},\begin{bmatrix}1\\ 2\\ -4\end{bmatrix},\begin{bmatrix}1\\ 0\\ -2\end{bmatrix}\right] \tag{4.11}$$

```
>  LnPeqns~(%,2);
```

$$[y-5x+3,2y+x-4,x-2] \tag{4.12}$$

```
> l1:=-x+1+3*y:l2:=2*x-13+3*y:pt:=LnPeqns(l1,l2,false):
  plt1:=RTPL([ l1,l2],range=[-1,8,-1,10]):
  plt2:=RTPT([2,0],l1,Txt=[typeset("line1 = ",l1),color="Magenta"],shift=1,fntos=12):
  plt3:=RTPT([2,10],l2,Txt=[typeset("line2 = ",l2),color="Blue"],shift=4,fntos=-12):
  plt4:=RTPT(pt,l1,Txt=[typeset("Intersection\n=",pt),color="Green"],shift=.7,fntos=45):
  plt5:=RTPP(pt):
  plots:-display(plt1,plt2,plt3,plt4,plt5,scaling =constrained)
```



See Also

<related help topic>, <related help topic>, <related help topic>, <Package Overview>